Communication and Contacts in Massively Interconnected Systems
Part 1: Rent’s Rule and Connectedness between People

By Pavel Barseghyan, PhD

Abstract

Communication and contacts between people are the heart of the organization of any human activity.

To account for this important factor in modern systems of management of human activities it is necessary to develop quantitative methods for the description of communication, contacts and connectivity between people.

Despite the obvious importance of this direction of research in organizational science it is still remains in the shadow of other equally important issues.

It is important to notice, that similar problems of quantitative description of connectedness of the elements and systems in electronics arose long time ago in the 60's of the last century.

Over the last 50 years in this area of electronics are obtained useful theoretical and practical results that can serve as a guide in organization science.

The first result in this direction of quantitative description of connectivity was the so-called Rent’s Rule [1]. Rent, an employee of IBM, empirically established a relation between the number of signal pins of logic blocks as a function of the number of elements in the block.

These studies continue to date in several ways, including the different applications of the Rent's Rule, generalizations of this rule for use in other areas, as well as the development of the theory of connectivity, which considers the communication and contacts between people as dynamic processes that take place in time and space [2, 3, 4].

Another area of research of communication and contacts between people is the Professor Thomas J. Allen’s research of MIT. In late 70s he discovered Allen Curve which is the exponential drop of frequency of communication between engineers as the distance between them increases [5]*.

This paper consists of three parts. The first part discusses the possibility of using the Rent's Rule to analyze communications and contacts in human or peopled systems.
The second part of the paper will address the elements of the theory of functions of connectedness and their applications for the analysis of human systems for the needs of organizational science.

The third part of the paper will discuss the links between differential equation of connectedness and Allen Curve.

Introduction

Communication and contacts between people is the core of any human activity, including program and project management, enterprise management and much more.

Together, all the different methods of communication and contacts between people determine their connectedness, which is the real core of the organization of work or undertaking of any size.

Accordingly, the considerations involved with the connectedness between people, must play a crucial role in modern methodologies of the organization of human work. It also means that the quantitative methods of organizational science, in turn, should take due account of the quantitative aspects of the problem of connectedness and contacts between people.

As for the complex and large-scale project works, because of their importance and greater propensity for failure, quantification of communication and contacts between their performers, as well as between development team and project environment is critical to success.

This type of quantitative understanding of the issues related to communication and contacts between people is also needed for a seamless interface with the other parts of modern management systems of programs and projects.

But this aspect of the management of human work, despite of their critical importance, continues to be overshadowed by other issues.

So far, the methods of analysis of this multifaceted issue of communication in human groups are reduced to the discussions on the properties of a simple combinatorial formula [6, 7]

\[ n_c = \frac{N(N-1)}{2} \]  

(1)

Here \( n_c \) is the number of communication links between \( N \) people.

Moreover, based on this simple combinatorial formula is made an erroneous conclusion that the number of contacts between people obeys the quadratic law.

In fact, this formula can have very limited use and it is more or less true for the small groups of people only, and then when it is expected that all members of the group are in contact with all the
others. This is an obvious idealization of the process of human interaction and which, even in small groups, can lead to false conclusions.

The main reason for this is that with increasing numbers of people, each individual has connections only with its immediate vicinity or environment, which is one of the signs of rationality of large systems. For example, in the design teams with a hierarchical structure is almost no connection between the groups of lower level. Studies show that this dependence is nearly linear, rather with an increase in the number of people asymptotically approaches the linear dependence [4].

Moreover, one of the objectives of the hierarchical structures of organizations is to reduce the number of unnecessary lower level contacts between people.

Therefore more realistic assessment of contacts between the humans needs new approaches to the mathematical modeling of their connectedness in design teams and other organizational structures, because the formula (1) overestimates the number of contacts.

**Characteristics of connectedness inside of the large human groups**

![Communication lines and contacts between people that live in regions Ω₁ and Ω − Ω₁](image)

Fig.1 Communication lines and contacts between people that live in regions Ω₁ and Ω − Ω₁

To start to analyze this problem using quantitative approaches suppose that the region Ω₁ inhabited by people evenly (Fig. 1).
Let's select in that region some area $\Omega$ around the point A on the boundary of the region and extend it as it is shown in Fig.1. Assume that the total number of people in the region $\Omega_1$ is $N_0$ and also contacts with the people living outside of the region $\Omega_1$ are not considered.

Fig.2 The number of contacts $n_{in}$ between people living in the areas $\Omega_1$ and $\Omega_1 - \Omega$ as a function of the number of people $N$ living in the area $\Omega$.

Consider the dependence of the number of contacts $n_{in}$ of the variable number of people $N$ living in the expanding area $\Omega$ with the people in the area $\Omega_1 - \Omega$ per unit time.

With a gradual increase in the area $S$ of $\Omega$ and, accordingly, a variable number of people $N$ living in this area, the number of contacts increases from zero and reaches its maximum at $N = \frac{N_0}{2}$. With the further growth of $N$, the number of contacts between people $n_{in}$ symmetrically will be reduced to zero at $N = N_0$.

Such a behavior is presented in Fig.2, where the human group under investigation consists of 100 people.

If we will consider systems containing different number of people $N_0$, the result will has the form as shown in Fig.3.

Here we have three human groups with number of people 60, 80 and 100 and the forth group (upper blue curve) that contains very large number of people (theoretically infinite number of people).
Fig. 3 The dynamics of communication lines and contacts between people depending on the total size of human groups

**Quantitative analysis of connectedness in large systems: Rent’s Rule**

Meanwhile, it should be noted that problems relating to the description of connectivity between the different parts of the system, in some other areas are being investigated for a long time. Thus, historically, the first attempt in this direction was the work of the employee of IBM Rent intended to study the number of signal pins of electronic blocks as the function of the number of elements in it, having a goal to select the appropriate connectors for blocks.

In 1960 E.F. Rent published an internal memorandum that contained a relationship between the number of external signal connections or terminals of a logic block $n_c$ and number of logic elements $N$ [1].

$$n_c = \alpha N^r$$  \hspace{1cm} (2)

Here, $\alpha$ is proportionality constant and $r$ is the Rent's exponent. The values of $\alpha$ and $r$ for the IBM computers were reported to be 2.5 and 0.6, respectively.
Rent’s Rule (2) is a mathematical model of connectedness of the groups of elements. In large systems it is a reflection of the behavior that is close to the upper blue curve in Fig.3. Coefficient $\alpha$ presents the total number of internal and external contacts for one person per unit time. Exponent $r$ indicates the ratio between internal and external contacts. The large exponent $r$ means a small number of internal contacts and a large number of external contacts. There are two limit cases $r = 0$ and $r = 1$.

For the case of the $r = 0$, all contacts are internal, and for the $r = 1$, all contacts are external.

Relationship (2) is presented in Fig.4 for typical values of the Rent’s exponent $r = 0.5$ (green curve), $r = 0.6$ (blue curve) and $r = 1.0$ (red curve).

The same rule can be applied for estimating the number of internal connections between elements [3]

$$n_{in} = \alpha (N - N^r) \quad (3)$$
Relationship (3) is presented in Fig.5 for typical values of the Rent’s exponent $r = 0.5$ (green curve), $r = 0.6$ (blue curve) and $r = 0$ (red curve).

Analysis shows that the Rent’s Rule in addition to electronics can also be applied in other areas too, including social networks, Internet and other Massively Interconnected systems [4].

According to this rule the number of internal and external connections of human groups can be described by the formulas (2) and (3).

![Graph showing interperson contacts $n_{in}$ as a function of the number of people $N$](image)

**Comparison of the formulas for contacts between people**

With the new formula (3) for evaluation of the number contacts within human groups one can compare it with the widely quoted formula (1). The joint plot of these curves is shown in Fig.6, where the red curve is presented the quadratic law of internal contacts and the other two curves represent the formula (3) for values of $r = 0.5$ (green curve) and $r = 0.6$ (blue curve).

Analysis of these curves indicates that the widely held view of the quadratic dependence of the number of contacts between people is not true. The reason for this is that with the increasing size of the group people begin to communicate with their immediate vicinity or environment only. This drastically reduces the number of contacts within human groups.
As noted above, the Rent's Rule in the form of (2) cannot serve as an adequate model of connectivity in the systems with a finite number of elements $N_0$, which is important for practical organization of human work.

Let's consider the derivation of the relationship for the finite systems on the basis of the Rent's Rule (2) and the principle of unequal division of connectivity fields [3].

Fig.6 Quadratic law (red curve) for the number of contacts between people is true for the relatively small human groups. This relationship is more of a nearly linear (green and blue curves). More precisely, with the increasing number of people in the group the relationship $n_{in}(N)$ is asymptotically becoming linear.

For this purpose, let's consider the group of $N_0$ elements of a system that are located on the $x$ axis and the connectivity in which obeys the Rent's Rule (Fig. 7).

This means that a group of $N_0$ elements will have $\alpha N_0^r$ contacts with elements located to the left of the point $0$ and to the right of the point $N_0$ on the $x$ axis. Half of these connections or contacts $\frac{\alpha}{2} N_0^r$ will be directed to the left, and the other half - in the right direction.
Let’s divide the group of elements $N_0$ into two subgroups with the number of elements $x$ and $N_0 - x$ which are also obeying the Rent’s Rule.

Correspondingly the number of contacts of these groups to the right side will be $\frac{\alpha}{2} x^r$ and $\frac{\alpha}{2} (N_0 - x)^r$.

The number of contacts in the subgroup $x$ of elements to the right side consists of two parts. The first part $C_x$ is the contacts with the subgroup of elements $N_0 - x$, and the second part $C_{x_2}$ is the contacts with the elements that are located to the right of the point $N_0$.

That means

$$C_x + C_{x_2} = \frac{\alpha}{2} x^r \quad (4)$$

In its turn the total number of contacts $\frac{\alpha}{2} N_0^r$ also consists of two parts: $\frac{\alpha}{2} (N_0 - x)^r$ and $C_{x_2}$ which means

$$\frac{\alpha}{2} N_0^r = \frac{\alpha}{2} (N_0 - x)^r + C_{x_2} \quad (5)$$

From here one can obtain

$$C_{x_2} = \frac{\alpha}{2} N_0^r - \frac{\alpha}{2} (N_0 - x)^r \quad (6)$$

Substituting expression (6) into the equality (4) one can obtain the number of contacts between the groups of elements $x$ and $N_0 - x$. 

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**Fig.7** Derivation of the Rent’s Rule for systems with finite number of elements
In fact this result is the analog of the Rent’s Rule for systems with finite number of elements considered above (Fig.2).

**Estimation of the number of contacts between remote groups of people**

Assume it is necessary to find the number of contacts between remote groups of people 1 and 3 (Fig. 8).

Based on the above method of determining the number of contacts between neighboring groups of people, one can also find the same number for the remote groups 1 and 3 [3].

Let’s denote the number of contacts between groups 1 and 3 through \(n_{1,3}\), between groups 2 and 3 through \(n_{2,3}\) and the number of contacts between the combined group 1+2 and 3 through \(n_{1+2,3}\). It is clear that there is a certain relationship between these numbers of contacts

\[ n_{1,3} = n_{1+2,3} - n_{2,3} \]  \hspace{1cm} (8)

Using expression (7) one can determine the number of contacts \(n_{1+2,3}\) and \(n_{2,3}\).

\[ n_{1+2,3} = \frac{\alpha}{2} \left[ (N_1 + N_2)^r + (N_3)^r - (N_1 + N_2 + N_3)^r \right] \] \hspace{1cm} (9)

\[ n_{2,3} = \frac{\alpha}{2} \left[ (N_2)^r + (N_3)^r - (N_2 + N_3)^r \right] \] \hspace{1cm} (10)

Substituting these values of \(n_{1+2,3}\) and \(n_{2,3}\) into the equality (8), one can obtain
Connectedness between people on a personal level

Expression (11) allows quantification of connectivity between people on a personal level. To do this, one can substitute into the formula \( N_i = 1 \) and \( N_j = 1 \) which can result the following expression

\[
n_{1,3} = \frac{\alpha}{2} \left[ (N_1 + N_2)' - (N_1 + N_2 + N_3)' - (N_2)' + (N_2 + N_3)' \right] \quad (11)
\]

Replacing in (12) \( N_2 \) with a new variable \( x \) one can obtain the connectivity function or the number of contacts in personal level \( m(x) \) and the distribution density function or the frequency of the number of contacts \( f(x) \) on a personal level.

\[
m(x) = \frac{\alpha}{2} \left[ 2(1 + x)' - (2 + x)' - (x)' \right] \quad (13)
\]

\[
f(x) = 2(1 + x)' - (2 + x)' - (x)' \quad (14)
\]
Fig. 9 presents the discrete view of the distribution function (14) for \( r = 0.6 \). This graph presents the frequency of contacts of a person with other people in his (her) encirclement.

It can be seen (Fig. 9) that if the unit of time is the working day, then during this period of time people in 50% of cases have contacts with the immediate neighbor, in 10% of cases they have contacts with the next neighbor (i.e., a neighbor of the second order), in 5% of cases - with the neighbor of the third order, and so on.

Also from expression (14) and Fig. 9 it is obvious that there must be a link between the Rent’s Rule and Allen Curve. This problem will be examined in the second and third parts of this paper.

**Applications of the Rent’s Rule for analysis of organizational hierarchies of human labor**

If we analyze the mechanisms of activities of enterprises and other large organizations, it appears that the demand for high productivity is crucial in determining the structure and the size of their units. In turn, the structure and the number of units are closely related to the characteristics of communication and contacts between people in these units.

This is also true for the hierarchical structure of large teams of the developers of complex projects. Rent's Rule can be applied for quantitative analysis and evaluation of the productivity of such development teams too [4].

The results obtained in this study suggest the possibility of solving the inverse problem, i.e., synthesis and optimization of organizational structure based on the requirement of maximum productivity.

**Conclusions**

1. Widely quoted combinatorial formula (1) to describe the communication between people is inadequate and should be replaced by more accurate models for the proper presentation of contacts and connections in process of human work.

2. Connectivity models that can provide an adequate description of the process of communication and contacts between people can be found in other areas of knowledge.

3. In particular, starting from the early 60's of the last century for estimation purposes of the parameters of electronic blocks people use an empirical relationship called Rent's Rule, which models the connectivity of electronic components and which can be successfully applied in organizational science.

4. Rent’s Rule can be used to assess such group characteristics of connectivity as the number of contacts between the neighboring and remote groups of people.
5. Rent rule can also be used for the analytical derivation of personal level connectivity characteristics.

6. It is necessary to establish relationships of a fundamental nature between group connectivity characteristics and personal level connectivity characteristics.

Concluding remarks and the main direction of research

It is possible to continue the list of useful applications of the Rent’s Rule. But it is also possible to list the problems that cannot be solved by the Rent’s Rule. More precisely, only a small minority of practical problems in the area of connectivity can be solved with the help of the Rent's Rule.

The main reason for that is the dynamical and variable character of connectivity in the real systems and the empirical Rent’s Rule is able to cover some static aspects of the whole problem only.

The history of science indicates that in similar situations the solution of the problems reduces to their description with the aid of differential equations that are able to reflect the dynamics and variability of the processes and phenomena under study [3, 8]

References


8. Pavel Barseghyan, Differential Equation of Connectedness for Massively Interconnected Systems

* I am grateful to John Goodpasture from whom I recently learned about the Allen Curve.
About the Author

Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 40 years’ experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia). Pavel’s publications can be found here: http://www.scribd.com/pbarseghyan and here: http://pavelbarseghyan.wordpress.com/.

Pavel can be contacted at pavelbarseghyan@yahoo.com