Elements of the Mathematical Theory of Human Systems

Part 1: Assessment of the Results of Human Actions and Activities Based on the Method of State Equations

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Abstract

Quantitative description of people's behavior should become the core of organizational science and a reliable basis for making various kinds of decisions for the problems that arise during the life of human society.

Whatever the area of human activity these decisions are made the center of the whole process is a certain **Human system**. It can be an individual, a group of people, a team of developers, an organization, a country, the whole world, etc.

The state of the **Human systems** can be stable or unstable, equilibrium or non equilibrium. In the equilibrium state, there is some balance between the system parameters (or variables), the reflection of which are the state equations of **Human systems**.

The combination of the equations of state with people's goals and the extreme principles of their behavior makes it possible to derive functional relationships between the parameters of the **Human systems** under study by analytical means that are common in physics and other quantitative sciences.

The universal nature of the method of state equations of **Human systems** allows us to use simple algebraic equations of state to adequately represent the behavior of vast variety of human activities, including the relations between the people, conflicts between them, international relations, and much more.

The heart of the mathematical theory of **Human systems** is the assessment of the results of human actions, because each action produces some result in the form of benefit or damage.

The first part of this paper is devoted to the assessment of the results of human actions and activities using the method of state equations of **Human systems**.

Introduction: The problems of constructing a quantitative theory of human systems

Traditional methods of managing people's activities, based on their experience and intuition, often fail because of the complexity and scale of the problems being solved and the limited time frame for their implementation.

The main reason for these difficulties is the limited ability of people who are responsible for making complex decisions on managing large-scale projects, entire sectors of the economy, state affairs, conflict situations between countries, and so on.

Even a simple observation of the course of human history clearly demonstrates that when difficulties arise in realizing their goals, people come up with appropriate tools that facilitate their work and increase their efficiency.

At present, exactly such a situation has arisen in a number of areas of human activity, where miscalculations in making decisions have catastrophic consequences [1, 2].

Solving this problem requires the creation of tools of a new type, helping humans in making complex decisions and which will naturally be associated with computer technologies.

But the use of computer technologies in any field implies a clear and accurate description of the relevant processes and phenomena, which in our case is related to the description of human actions and activities and mathematical modeling of human behavior.

From here the fundamental need to create mathematical models of people's actions and activities [3].

Since any human activity is a sequence of people's actions, the first task of modeling their behavior and decision-making process is the creation of mathematical models of people's actions.

Each action is the fulfillment of the next requirement of life by a person who, in order to achieve this goal, spends effort, time, knowledge and skills.

Besides, each action has its own mathematical equation, which reflects the balance or equilibrium between the requirements of life and the ability of man [3].

State equation of an arbitrary human action (or activity) is a consequence of the balance between the need to commit actions (or activity) by a person and his/her ability to bring them to completion

To ensure the normal life process of any human system, a balance or equilibrium is needed between the big and small problems that arise and accompany a person during the life process on the one hand and the ability of people to solve these problems on the other hand.

The life of people is a sequence of actions, each of which can be characterized by the magnitude (or size, or scale) W and difficulty of execution D.

Each action of people is also characterized by its cumulative complexity C_d, determined by its size W and difficulty D by the following expression

$$C_d = W * D \qquad (1)$$

People, depending on their abilities and having different levels of skills and knowledge, can overcome the complexity of the action C_d, partially or completely, while ensuring different performance qualitiy Q. This means that C_d can be defined as $C_d = Q * W * D$.

Thus, a person, having productivity P (which is a reflection of his/her skills and knowledge) and consuming effort E, can overcome certain portion C_s of the complexity C_d defined as

$$C_s = E * P. \qquad (2)$$

That is, on the one hand, to ensure the normal course of the life process, there is a need for people's actions, and on the other hand, as a response to the demands of life, we have people's real actions, which are a reflection of their skills and abilities.

If we draw a parallel with the economic theory of supply and demand, then the need for human actions is the demand of the life process, and the knowledge and skills of people fulfill the role of the supply.

As it was said above, the successful execution of actions by people implies some balance or equlibrium between the complexity of the action C_d and the ability of people C_s to overcome this complexity.

The simplest case of this balance is the equality of the indicated complexities, that is $C_s = C_d$, or

$$E * P = W * D, \qquad (3)$$

which is the condition for the equilibrium of life and activity of people.

People's effort is defined as

$$E = N * T, \tag{4}$$

where N - is the number of people participating in the action (or activity), and T - is the duration of the people's efforts.

Thus, the condition for the equilibrium of people's activities will take the form:

$$N * T * P = W * D. \tag{5}$$

The meaning of this equality is that the group of people consisting of N person, acting with productivity or efficiency P for a period of time T, can carry out an action or activity that has magnitude or size W and difficulty D.

In the case of one person, the equation of state (5) is transformed into the following equation

$$T * P = W * D. \tag{6}$$

This also means that if we consider people's activities as a system, then at a high level, such a system can be described by five parameters or system variables N, T, P, W and D.

Between these parameters or system variables there are functional relationships of a fundamental nature. For example, increasing the difficulty D of actions and activities leads to a decrease in their performance P.

Also, a decrease in the planned duration T of work leads to an increase in the number of performers N, which in turn leads to a decrease in people's productivity P due to an increase in the time of contacts between people for communication and coordination of their efforts, and so on.

Parameter P that appears in the equation of state is the efficiency of the activity of a person (or human system), which in the simplest case is equal to the number of successfully performed actions by a person (or human system).

This parameter has a functional connection with the level of motivation M and the maximum possible value of the effectiveness P_{Max} of humans (or human systems) activity which has the following simple form:

$$P = M * P_{Max} \tag{7}$$

Taking into account the expression (7) and human's performance quality Q, the equations of state (3), (5) and (6) take the following form

$$E * M * P_{Max} = Q * W * D, \tag{8}$$

$$N*T*M*P_{Max} = Q*W*D, \quad (9)$$

$$T*M*P_{Max} = Q*W*D. \quad (10)$$

Equations of state (8), (9) and (10) can be expanded and deepened by taking into account intensity of their activity I_{Max}, intensity of errors a made by people and average time t_a for correcting one error [5].

$$E*M*I_{Max} = Q*W*D(1+at_a), \quad (11)$$

$$N*T*M*I_{Max} = Q*W*D(1+at_a), \quad (12)$$

$$T*M*I_{Max} = Q*W*D(1+at_a). \quad (13)$$

The meaning of the typical equations of state (12) of a human system can be illustrated as follows: the group of people consisting of N person, having motivation M and acting with intensity I_{Max} for a period of time T, with quality Q and errors at_a can carry out an action or activity that has magnitude or size W and difficulty D.

Each action and activity of a person has some result, which can be estimated using the equation of state

Any kind of human activity requires effort, knowledge and skills that allow people to pursue certain goals, overcome the complexity of the next life requirement, having obtained a result that has a certain level of quality.

Different types of activities of people give different results: the manager makes decisions and works on their implementation, the engineer programmer produces code, the successful activity of the businessman produces wealth, the farmer - vegetables and fruits, the successful activity of various branches of power produces justice, ideologists - concepts and ideas, and so on.

This means that in order to complete the quantitative representation of people's behavior, it is also necessary to take into account the results of their activities in the equations of state, including the benefits and losses that result from certain types of activities.

Analysis of the equation of state of the human system shows that the role of the bridge between people's activities and their results can serve the volume W of their activities.

The reason for such a transition is that people usually carry out some kind of activity in order to benefit from it, be it financial or material gain, purely moral satisfaction or achievement of far-reaching political goals, etc.

In addition, people expand their activities in order to increase the expected generalized benefits from that.

That is, it is implied that an increase in the volume W of activity of the human system will lead to an increase in its result R, which in the simplest linear case can be expressed by the equation

$$R = k_R W, \qquad \text{or} \qquad W = \frac{1}{k_R} R. \tag{14}$$

Here coefficient k_R shows the efficiency of the transition from the volume of activity W to its result R, and this coefficient can be either positive or negative, which is associated with possible particular cases of positive and negative results from any activity of humans.

Naturally, between the volume of activity W and its result R, besides a linear one, there can be a multitude of other nonlinear functional dependencies, but this simple linear case is aimed at a clear and transparent illustration of the meaning and effectiveness of the state equation method for quantitative representation of human behavior.

So, if we substitute the expression (14) into equations (11), (12) and (13), then, respectively, we get:

$$k_R * E * M * I_{Max} = Q * R * D(1 + at_a), \tag{15}$$

$$k_R * N * T * M * I_{Max} = Q * R * D(1 + at_a), \tag{16}$$

and

$$k_R * T * M * I_{Max} = Q * R * D(1 + at_a) \, . \tag{17}$$

From this, defining result R from the equation (15) for the generalized result of people's activity, we will have

$$R = \frac{k_R * E * M * I_{Max}}{Q * D(1 + at_a)}. \tag{18}$$

To assess the results of the group of N people from the expression (16) we have

$$R = \frac{k_R * N * T * M * I_{Max}}{Q * D(1 + at_a)}. \qquad (19)$$

The results of one person's activity can be expressed by the equation of state

$$R = \frac{k_R * T * M * I_{Max}}{Q * D(1 + at_a)}. \qquad (20)$$

State equations for the rate of accumulation of the results of human activities

If we take into account the fact that the ratio $\frac{R}{T}$ is the average rate of accumulation or growth of the result of human activity, this will allow to extend the possibilities of the equations of state of human systems indirectly taking into account also the time factor in them.

On this basis, from the expressions (19) and (20) one can obtain new equations of state, which also contain the rates of people's activity

$$V_R = \frac{k_R * N * M * I_{Max}}{Q * D(1 + at_a)}, \qquad (21)$$

and

$$V_R = \frac{k_R * M * I_{Max}}{Q * D(1 + at_a)}. \qquad (22)$$

Profit or income as a result of human activity: Quantitative model

Continuing the reasoning and using the resulting equations of state, we can establish functional relationships between parameters that reflect the activities of people and their generalized results.

The point is that the same action or activity may have different outcomes, including material, moral, financial, political, etc. results, each with its generalized value and effectiveness.

For example, if a person produces some product, the amount of which is proportional to the value W and from which the person receives income B, then mathematically this can be represented in the following simple linear form

$$B = k_B W \qquad (23)$$

Here the coefficient k_B indicates the measure of the effectiveness of the transformation of human activity into income, which in turn can depend on the characteristics of the person (or a human system), and the nature of the environment in which he/she (or a human system) acts.

This circumstance, in turn, can become a new source for many new branchings of the equation of state, if we construct the appropriate bridges between the coefficient k_B and characteristics of people (or a human system) and environment.

In any case, expression (23) allows us to proceed to a quantitative description of the activity of humans of a fundamentally new character, where along with other parameters of the human system, the resulting or expected generalized income B also participates.

Substituting expression (23) into the equations of state of human systems (8) and (12), respectively, we get

$$k_B * N * M * T * P_{Max} = Q * B * D. \qquad (24)$$

and

$$k_B * N * T * M * I_{Max} = Q * B * D(1 + at_a). \qquad (25)$$

Hence, in order to estimate the expected income from human activities, we will have

$$B = \frac{k_B * N * M * T * P_{Max}}{Q * D}, \qquad (26)$$

and

$$B = \frac{k_B * N * M * T * I_{Max}}{Q * D(1 + at_a)}. \qquad (27)$$

The meaning of these expressions is that the generalized income B received from any activity is proportional to the effectiveness k_B of the implementation of the results of this activity and the duration T of activity, people's motivation M and their professional qualities P_{Max} or intensity of activity I_{Max} and inversely proportional to the requirements for the quality of results Q, the difficulty of creating or generating results D, and also made errors at_a.

Interpretation of the equations of state of human systems as bridges between resources and results of human actions and activities

According to a more general interpretation of the meaning of the state equation of human systems, it is the link between the resources on which people rely in their activities and the results of this activity (Fig. 1).

Based on this interpretation of the meaning of the state equations of human systems, it is possible to expand and deepen these equations both in the direction of the resources necessary for people's activities and in the direction of its results obtained as a result of this activity.

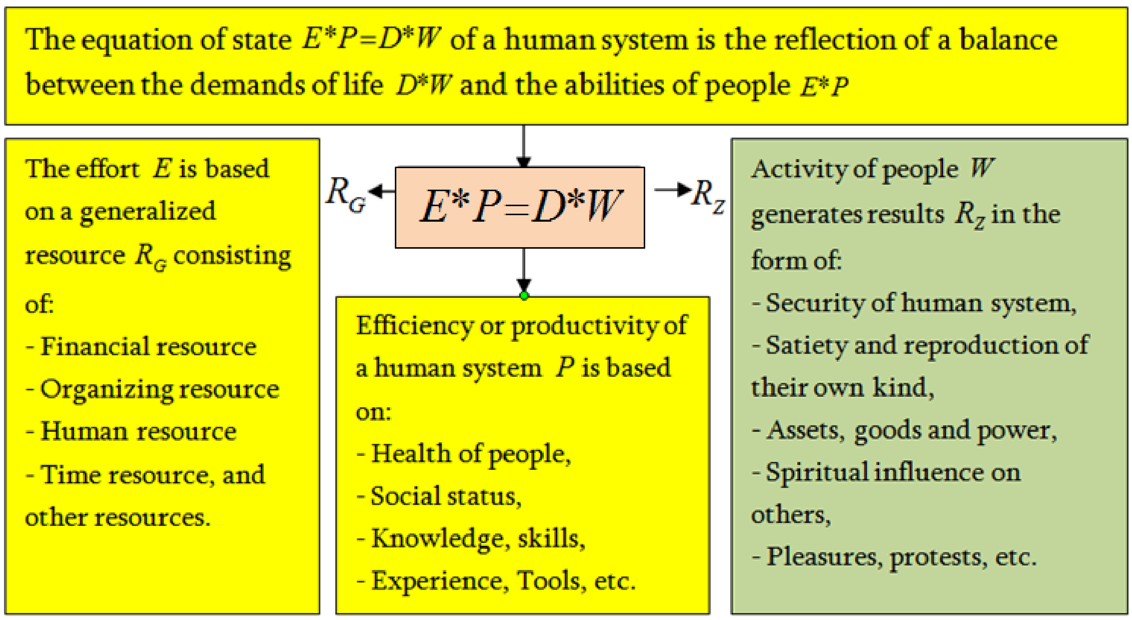

Fig.1 The activity of people relies on resources and produces results, the reflection of which are the equations of state of human systems

Thus, it is possible to build a generalized mathematical model of the activity of human systems, in which both the capabilities of people, and the various resources on which they rely in their activities, and the diverse results of human activity are presented.

From the mathematical point of view, the construction of a generalized activity model of people is based on the fact that the parameters of the initial equation of state have functional links with the resources of the human system, and with the results of its activities.

Above, for expanding the possibilities of the equations of the state of human systems, the relationship between the volume of activity W and the results R of this activity was used.

To establish the quantitative links of the activity of human systems with the necessary resources for this, one can use the duration of activity T .

The idea of establishing this functional relationship is based on the statement that each time interval is equivalent to spending some amount of resource. This means that between the duration of the activity of human systems and the amount of the resource consumed there is a simple functional dependence

$$\mathrm{Re} = k_{RT} T \quad \text{or} \quad T = \frac{\mathrm{Re}}{k_{RT}}, \qquad (28)$$

where the value k_{RT} - is the rate at which the resource is used.

Substituting expression (28) into the equation of state (16), one can obtain

$$k_R * N * \frac{\mathrm{Re}}{k_{RT}} * M * I_{Max} = Q * R * D(1 + at_a)\,. \qquad (29)$$

Hence, to evaluate the results of the activity of the human system, we obtain the following expression

$$R = \frac{k_R * N * M * I_{Max} * \mathrm{Re}}{k_{RT} * Q * D(1 + at_a)}, \qquad (30)$$

which combines the parameters of this system, namely the number of people N, their motivation M, the intensity of activity I_{Max}, the degree of rationality of people's activities k_R, the quality Q and difficulty D of this activity, the intensity of people's errors at_a with the resources cunsumed Re and the results R of their activities.

In principle, this expression can be extended to any human system, since all of them use resources in their activities and achieve certain results using the skills and knowledge of humans.

The resulting formula (30) can also be considered as one of the possible forms of the production function of an arbitrary human system, which is analogous to production functions from economic science [6].

But unlike economic production functions, their new form contains parameters that also reflect the characteristics of people, reflecting their skills and knowledge, as well as the errors that accompany their activities.

Conclusions

1. The method of state equations enables to formalize the description of people's lives in many ways;
2. The equation of the human system unites the resources, knowledge and skills of people and the results of their activities;

3. The method of state equations is a universal tool and a reliable foundation for mathematical sociology and geopolitical sciences, which allows in a deductive way to obtain functional relationships between parameters that reflect the social and organizational side of people's lives;

4. Expanding and deepening the equations of state in different directions allow us to consider people's lives as a system of tradeoffs of various aspects of life process and different requirements;

5. At the same time, the equations of state allow us to establish functional dependencies between various aspects of people's lives, which at first glance are not related to each other. Thus, the method of equations of state becomes a means of detecting and synthesizing unknown aspects of human life;

6. On the basis of mathematical models constructed on the basis of the method of state equations, it is possible to create software tools for making decisions necessary for the management and control of human systems.

Future research

The activity of human systems of any complexity is a sequence of actions of a different nature that can be mathematically represented as a system of state equations;

In addition, different organizational structures of human systems and diverse geopolitical problems can also be represented as a system of state equations.

These and other issues will be discussed in the second part of the article.

The third part of the article is devoted to the construction and analysis of mathematical models of cooperation and confrontation between human systems based on the method of state equations.

In particular, within the framework of mathematical models of cooperation and confrontation, problems of formation and disintegration of coalitions and unions of human systems are considered.

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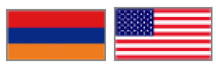

Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia). Pavel's publications can be found here: http://www.scribd.com/pbarseghyan and here: http://pavelbarseghyan.wordpress.com/. Pavel can be contacted at terbpl@gmail.com