Elements of the Mathematical Theory of Human Systems, Part 4: Quantitative interpretation of the victory, defeat and concessions of human systems by the method of state equations

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Abstract

The mathematical theory of human systems has many applications in the fields of analysis, synthesis and optimization of confrontations and conflicts between the groups of people of different scales.

The clash between human systems arises from a conflict of interest, be it the usual conflict of interests in people's daily lives or a serious clash of interests in international relations, or a clash of civilizations.

The purpose of the article is to show that the problems of quantitative description and analysis of conflicts between human systems can be reduced to the method of state equations.

According to this method, the activities of each party to the conflict can be represented by the equation of state, on the basis of which the benefits and losses obtained from the activities of the parties are evaluated.

In the case of significant differences between the benefits and losses of the conflicting parties, the balance of power between them can be violated, which can lead to non-equilibrium phenomena such as the victory of one of the parties.

In the work on the basis of quantitative assessments of the benefits and losses of the conflicting parties, quantitative interpretations of the victories and defeats of people are also given.

The methods discussed in the article may have many practical applications, including the analysis of different types of competition between human systems, as well as assessments and predictions of the results of the conflict between countries and their various blocs and alliances.

Key words: Human systems, mathematical theory, state equations, systems theory, equilibrium, non-equilibrium, benefits, change management, losses, victory, defeat, concessions, conflicts.

Introduction

For mathematical modeling and simulation of confrontations and conflicts between human systems, it is necessary to have an adequate quantitative description of their activities and relations among themselves.

The mathematical theory of human systems, the axis of which is a quantitative description of the actions and activities of people by means of the equations of state, is also suitable for studying and managing confrontations and conflicts between different groups of people.

In different cases, the conflicting parties may be negotiators in business and diplomacy or participants in a scientific debate whose opinions differ on the issues under discussion, or the equation of state may partially describe the relationship between the client and the seller, or between the customer and vendor, etc.

In addition, in the context of the further effective penetration of quantitative methods into the sphere of human systems management, the symbiosis of the possibilities of behavioral models of people's life based on the equations of state [1] and the basic propositions of systems theory is very important [2, 3].

Such symbiosis allows, along with traditional qualitative methods of studying human systems, to create new scientific methods for analyzing, synthesizing and optimizing the behavior and activities of people on the basis of quantitative approaches of a fundamental nature.

The essence of quantitative methods of analyzing the behavior and activities of people, based on the equations of the state of human systems, is as follows.

Each human system can be quantitatively described by an equation of state that incorporates the needed resources for people's activities, their knowledge, skills and tools used by them and the results of their activities.

In this sense, the purpose of quantitative analysis of the behavior and activity of human systems is to evaluate and predict the results of their activities in the form of gain and losses based on the equation of state and the characteristic data of the system (the number and motivation of people, the effectiveness of their activities and tools used, etc.).

In the case of confrontation and conflicts of human systems, the problem of analyzing their behavior and activities is to assess the results of their clash in the form of victory (or gain) and defeat (or loss), having the data of the conflicting parties.

The problem of the synthesis of human systems is that by having the desired results of its behavior and activity, by inverse calculation on the basis of the equations of state, find those values of the system parameters that can provide the given desired result.

In the case of confrontation and conflicts between human systems, the problem of synthesis in planning the behavior and activity of one of the parties is to use the method of state equations to find those values of the system parameters with which it will be possible to defeat the potential enemy or, at least, not to be defeated by him.

Since any synthesis problem has a lot of solutions, the goal of optimal synthesis or optimization of systems is to choose from this set of solutions those that are more preferable in terms of price, quality, risk and feasibility.

The method of the equations of state has a deterministic character and describes the problem of analysis, synthesis and optimization at the level of average values of the parameters of human systems.

But, as is known, mathematical modeling of systems at the level of mean values of parameters can be considered as a first approximation, which does not always lead to comprehensively grounded solutions to problems [4].

From this point of view, for a more detailed consideration of the problem, the parameters in the equations of state of human systems can be divided into three groups:

- Parameters related to the presentation of resources for the normal activities of people and their supply chains;
- Parameters related to characterization of people and different scale of human groups, including their professional qualities and capabilities, as well as their tools and system's infrastructure;
- Parameters associated with the characterization and interpretation of the results of the activities of human systems.

It is obvious that these parameters have both deterministic and random components, which put forward the problem of probabilistic interpretation of the equations of state of human systems.

In the case of confrontations and conflicts of human systems, this means that estimates and forecasts of potential benefits and losses of conflicting parties must have two levels of model representation: deterministic and probabilistic.

Deterministic models of behavior and functioning of the human system serve as the averaged skeleton, to which a layer of probabilistic estimation and forecasting mechanisms are added at the second level of modeling.

In this paper, mathematical models of a deterministic nature are considered, related to the quantitative representation of confrontation and conflicts of human systems.

Equations of state of the conflicting parties

Consider the problem of a quantitative description of the conflict between two human systems, the first of which within the time interval T reaches a preponderance, and then a victory over the other side.

Consider a quantitative description of this phenomenon using the equations of state of these conflicting systems.

The first side of the conflict uses part of the total time T equal to T_{12} for the activities associated with the second party, in which the number of participated people is N_{12} and the effectiveness of their actions is P_{12}. The overall effectiveness of these people's actions will be

$$H_{12} = N_{12} * P_{12}, \qquad (1)$$

with the help of which they will overcome the difficulties D_{12} and scope W_{12} of activities associated with confrontation.

This means that the actions or activities of the first party can be described by the following equation of state [1].

$$T_{12} * N_{12} * P_{12} = W_{12} * D_{12}. \quad (2)$$

The first party by its activity produces a result or pressure on the second side, which is expressed by the following linear formula

$$R_{12} = k_{R12} W_{12}, \text{ or } \quad W_{12} = \frac{R_{12}}{k_{R12}}, \quad (3)$$

where the coefficient k_{R12} is the result produced by the unit activity W_{12}.

This coefficient, which illustrates the effectiveness of the transition from human activity to its results, can be interpreted as a measure of the level of human skills, perfection of the tools, means or weapons used during this activity.

Under the words "means, tools or weapons" in one case one can understand a real weapon such as a rocket or a tank, in other cases it can be money, human speech, the power of art, and much more.

Substituting the value W_{12} from (3) into equation (2), one can obtain:

$$k_{R12} * T_{12} * N_{12} * P_{12} = R_{12} * D_{12}. \quad (4)$$

Hence the pressure of the first side of the conflict on the second side will be determined by the following expression, which is also the equation of state

$$R_{12} = \frac{k_{R12} * T_{12} * N_{12} * P_{12}}{D_{12}}. \quad (5)$$

Similarly, the pressure of the second side of the conflict on the first side will look like this:

$$R_{21} = \frac{k_{R21} * T_{21} * N_{21} * P_{21}}{D_{21}}, \quad (6)$$

where the parameters have the same meaning as in (5), but relate only to the influence of the second side of confrontation on the first party.

In any conflict, the motivation of the parties and their errors are crucial, so it is advisable to take these parameters into account in equations (5) and (6).

For this, instead of efficiencies P_{12} and P_{21}, one can substitute their expressions from [1], that is:

$$P_{12} = \frac{M_{12} * I_{12}}{(1 + a_{12} t_{12a})} \quad (7)$$

and
$$P_{21} = \frac{M_{21} * I_{21}}{(1 + a_{21} t_{21a})} \qquad (8)$$

As a result of these substitutions, we will get

$$R_{12} = \frac{k_{R12} * T_{12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12} t_{12a})}. \qquad (9)$$

$$R_{21} = \frac{k_{R21} * T_{21} * N_{21} * M_{21} * I_{21}}{D_{21}(1 + a_{21} t_{21a})}. \qquad (10)$$

The effectiveness of activities of the conflicting parties can be assessed using various combinations of parameters of the equations of state, including the pressures generated by the parties per unit time $r_{T12} = \frac{R_{12}}{T_{12}}$ and $r_{T21} = \frac{R_{21}}{T_{21}}$, or pressures per person $r_{N12} = \frac{R_{12}}{N_{12}}$ and $r_{N21} = \frac{R_{21}}{N_{21}}$, or pressures per person per unit time $r_{TN12} = \frac{R_{12}}{T_{12} N_{12}}$ and $r_{TN21} = \frac{R_{21}}{T_{21} N_{21}}$, and so on.

The meaning of the obtained results and their possible generalizations

All the results obtained above are, in fact, the equations of state of the human system, which contain a large number of explicit and implicit functional relationships between the various parameters of the system.

For brevity, consider the interpretation of only one of the resulting equations of state, namely, equation (9), that is, the pressure of the first human system on the second.

So the meaning of the equation (9) is that the human system consisting of N_{12} individuals, acting with motivation M_{12}, intensity I_{12} and making $a_{12} t_{12a}$ errors and with efficiency k_{R12} overcoming difficulties D_{12} during the period of time T_{12} generates or produces a result R_{12}, which in our case is a generalized pressure on the opposite side of confrontation.

It is obvious that there are various functional dependencies between the parameters of the equations of state under consideration, therefore, changes in one or more of these parameters will lead to different nonlinear changes in other parameters of the equations.

Since the life of any human system is a sequence of changes that may be uncontrolled or controlled and intentional, subsequent generalizations of the methodology based on the equations of state should be related to methods for analyzing and managing these changes.

This area of research, known as change management, by using the method of state equations can get great potential and good opportunities for the development of new approaches in the management of human systems.

In particular, from the point of view of change management, it is important to know the structure of the pressure on the parties of confrontation and its sensitivity, depending on changes in different parameters of human systems.

In this sense, considerable useful information can be obtained from the expression for the pressure change on the opposite side as a function of changes in different parameters, which for the pressure R_{12} will have the following form

$$\Delta R_{12} = \frac{\partial R_{12}}{\partial k_{R12}} \Delta k_{R12} + \frac{\partial R_{12}}{\partial T_{12}} \Delta T_{12} + \frac{\partial R_{12}}{\partial N_{12}} \Delta N_{12} + \frac{\partial R_{12}}{\partial M_{12}} \Delta M_{12} +$$

$$+ \frac{\partial R_{12}}{\partial I_{12}} \Delta I_{12} + \frac{\partial R_{12}}{\partial D_{12}} \Delta D_{12} + \frac{\partial R_{12}}{\partial (a_{12} t_{12a})} \Delta (a_{12} t_{12a}) \qquad (11)$$

This expression establishes functional relationships between linear variation ΔR_{12} of the pressure R_{12} and linear changes of other parameters whose coefficients are partial derivatives of R_{12} with respect to these parameters.

This allows us to use expression (11) for analysis and synthesis of equilibrium and non-equilibrium modes of interaction of human systems.

Moreover, for the purposes of more detailed and in-depth analysis and synthesis of relations between different countries, one can divide the general flow of their mutual actions into political, economic, diplomatic, military and other sub-flows.

Each of these sub-flows of the parties' actions will have its own equation of state, which is organically related to the equations of states of the other action sub-flows, which allows us to view the behavior of the human system as a whole.

The resulting equations of state and expressions show that the mutual pressures of opposing sides can cause various conflicts, ranging from the ordinary discussions of problems to peaceful competition between rivals, as well as to local and global political, diplomatic and military clashes.

At the same time, between the opposing sides there are possible a variety of equilibrium and non-equilibrium states that can lead to partial or complete victories and benefits of the parties, or their defeats and losses.

Dependence of the parameters of the equations of state on time

Mathematical models of the confrontation of human systems can also be generalized to cases where the parameters of the equations of state are time-dependent functions.

For that purpose, consider equation (4) for the period ΔT during which the first side of the conflict accumulates pressure ΔR_{12} on the second party.

$$k_{R12} * \Delta T_{12} * N_{12} * P_{12} = \Delta R_{12} * D_{12} \qquad (12)$$

From here one can get

$$\frac{\Delta R_{12}}{\Delta T_{12}} = \frac{k_{R12} * N_{12} * P_{12}}{D_{12}} \,, \qquad (13)$$

or, passing to the limit, we obtain a differential equation for accumulating pressure on the second side of the confrontation

$$\frac{dR_{12}}{dT_{12}} = \frac{k_{R12} * N_{12} * P_{12}}{D_{12}} \,, \qquad (14)$$

the solution of which, under the initial condition $R_{12T=0} = R_{120}$, will be

$$R_{12} = R_{120} + \frac{k_{R12} * N_{12} * P_{12}}{D_{12}} T_{12} \,. \qquad (15)$$

This solution takes into account the pressure accumulated by the first party of conflict on the second party prior to the observed period of time T .

Taking into account expression (7), the solution of the differential equation (15) takes the following form

$$R_{12} = R_{120} + \frac{k_{R12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12} t_{12a})} T_{12} \,, \qquad (16)$$

which also includes the prehistory of results R_{120} before the time period T .

Similarly, the second side of the conflict will exert pressure on the first party, which will be expressed by the following equation of state

$$R_{21} = R_{210} + \frac{k_{R21} * N_{21} * M_{21} * I_{21}}{D_{21}(1 + a_{21} t_{21a})} T_{21} \,. \qquad (17)$$

A multi-faceted equilibrium regime between the opposing sides

The equilibrium regime between interacting human systems is possible in a number of cases:

1. When, as a result of their activities, mutual R_{12} and R_{21} pressure values are constant and do not change over time,
2. When, as a result of their activity, the mutual variable R_{12} and R_{21} pressure values are equal to each other at any time,

3. When the changes ΔR_{12} and ΔR_{21} of the pressure values are equal to each other at any time.

In the first case, the parameters of each side of the conflict can be changed, but in such a way that the pressure values R_{12} and R_{21} remain constant, the condition that for pressure R_{12} will have the following form:

$$R_{12} = \frac{k_{R12} * T_{12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12} t_{12a})} = Const. \tag{18}$$

This condition implies that some of the parameters of expression (18) can undergo such changes, in which the value R_{12} remains unchanged.

This can be achieved through simultaneous changes in the number of people, their motivation, intensity of their activities and with the help of other changes.

In this case, it is necessary to take into account the fact that changes in the values of certain parameters of the equations of state are accompanied by spontaneous nonlinear changes in the values of other parameters.

If, for example, for some reason the difficulty D_{12} of actions increases, this will lead to a decrease in intensity I_{12}, which in turn will require an increase in the number of people N_{12}, which in its turn will complicate the problems of coordination of people's actions and their communication.

In the second case, the equality condition $R_{12} = R_{21}$ using the equations of state (9) and (10) leads to the following combined equation of the state:

$$\frac{k_{R12} * T_{12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12} t_{12a})} = \frac{k_{R21} * T_{21} * N_{21} * M_{21} * I_{21}}{D_{21}(1 + a_{21} t_{21a})}, \tag{19}$$

where the left and right sides of the equation are variable quantities.

In a more general case, using equations (16) and (17), one can obtain a new equation of combined state that takes into account the initial accumulations of the mutual pressures R_{120} and R_{210} of human systems

$$R_{120} + \frac{k_{R12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12} t_{12a})} T_{12} = R_{210} + \frac{k_{R21} * N_{21} * M_{21} * I_{21}}{D_{21}(1 + a_{21} t_{21a})} T_{21}. \tag{20}$$

The resulting equations (19) and (20) reflect the dynamic balance between the conflicting parties, which allows to manage changes in people's lives by quantitative means.

In practical terms, the equation of state (20) will help to find answers to the following questions.

What will happen if the first party of the conflict as a result of purposeful actions makes serious changes in pressure on the opposite side, and what can be the consequences in terms of internal changes of the first party and what will be the possible reaction of the second party?

Naturally, both internal and external changes of the conflicting parties will depend on whether the pressure change has a positive or negative impact on the second side and, in addition, what objectives are pursued by the parties of the conflict.

Joint solutions of the equations of state and objective functions, reflecting the behavior of human systems, provide answers to some of the above problems [4, 5].

In the third case, the problem of preserving parity between conflicting parties leads to an analysis and possible solutions of the equation $\Delta R_{12} = \Delta R_{21}$, which can be described in detail by a combination of equation (11) with the objective functions of the parties.

The diversity of non-equilibrium regimes between opposing sides: Quantitative interpretation of the victory and defeat of human systems

Consider the non-equilibrium states of the relationships between human systems that can arise for a variety of reasons, including their different civilizational opportunities, or because of the differences between their ideologies and many other reasons.

Quantitative description of the non-equilibrium states of relations between human systems can be represented by the following inequalities,

$$R_{12} > R_{21} \text{ or } R_{12} < R_{21}, \qquad (21)$$

which reflect the cases when the generalized pressure of one of the opposing sides exceeds the opposite pressure.

Let us consider in more detail the inequality $R_{12} > R_{21}$, which with the help of expressions (9) and (10) will look like this:

$$\frac{k_{R12} * T_{12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12}t_{12a})} > \frac{k_{R21} * T_{21} * N_{21} * M_{21} * I_{21}}{D_{21}(1 + a_{21}t_{21a})}, \qquad (22)$$

or taking into account the initial accumulation of mutual pressures R_{120} and R_{210} between human systems, we will have:

$$R_{120} + \frac{k_{R12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12}t_{12a})} T_{12} > R_{210} + \frac{k_{R21} * N_{21} * M_{21} * I_{21}}{D_{21}(1 + a_{21}t_{21a})} T_{21} \,. \qquad (23)$$

Consider the phenomenon of transition from a non-equilibrium state $R_{12} > R_{21}$ between human systems to an equilibrium state, which is possible if the first party has any tangible or intangible benefit from the second side, which can be characterized as the victory of the first side of the confrontation, or the defeat of the second party.

In other words, the transition to a new equilibrium between human systems will require the second side to make certain concessions of a territorial, financial or reputational nature, the size of which, with a linear approximation, will be proportional to the difference between the mutual pressures R_{12} - R_{21}.

That is, the size of the concession of the second side with linear approximation will be

$$Z_{12} = k_{Z21}(R_{12} - R_{21}), \qquad (24)$$

where the coefficient k_{Z21} is the generalized price of the concession, which is equivalent to the unit pressure difference of the opposing sides.

Substituting in the expression (24) the pressure values R_{12} and R_{21}, we obtain

$$Z_{12} = k_{Z21}\left(\frac{k_{R12} * T_{12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12}t_{12a})} - \frac{k_{R21} * T_{21} * N_{21} * M_{21} * I_{21}}{D_{21}(1 + a_{21}t_{21a})}\right), \qquad (25)$$

or $Z_{12} = k_{Z21}(R_{120} + \frac{k_{R12} * N_{12} * M_{12} * I_{12}}{D_{12}(1 + a_{12}t_{12a})} T_{12} - R_{210} - \frac{k_{R21} * N_{21} * M_{21} * I_{21}}{D_{21}(1 + a_{21}t_{21a})} T_{21})$. (26)

These expressions show either the generalized price of winning as a result of the victory of the first side of the confrontation over the second party, or the generalized price of the defeat of the second party. These expressions can be widely used in the field of conflict analysis.

Along with the linear form of concessions (24), which is adequate for the cases of relatively simple confrontations of human systems, nonlinear forms of the dependence of the concession on the difference between the mutual pressures of the parties are also important.

The necessity of such forms of approximation arises in acute conflict situations, when the parties act in areas close to the upper limits of their possibilities [6].

Future research

The results obtained in this article and the methodology of the analysis of conflicts of human systems have considerable potential and are applicable to the solution of the following problems.

- Quantitative analysis and synthesis of confrontations and conflicts of human systems in the form of coalitions, blocs and various unions of groups of people of different scale;
- Development of multilateral and detailed mathematical models of confrontations and conflicts through sectoral equations of states of different aspects of the parties' activities;

- Quantitative models of the relationships between vendors and customers, as human systems that are interested in the success of the project, but at the same time pursue different goals;

- Mathematical models of simple conflicts of everyday life between people in the field of linear difficulties and in a nonlinear range of difficulties at the upper limits of human capabilities;

- Nonlinear opposition of a weak human system with a strong human system in the field of linear activity of the latter;

- Construction of deterministic models of interactions between similar in nature and geographically close conflicts by the method of state equations;

- Development of probabilistic models of confrontations and conflicts of human systems and study of their interactions, superposition and nonlinear resonances using the method of state equations;

Conclusions

1. Considering victory or defeat as a result of actions and activities of people, the method of equations of state can be used for their quantitative representation;

2. Confrontations and conflicts between human systems can be quantified using the equations of state and interpreted as mutual pressures of the generalized nature of the conflicting parties;

3. With the help of appropriate interpretations of the parameters of state equations, the mutual generalized pressure between people can be represented as a function of the capabilities of people and resources necessary for their activities;

4. The study of confrontation and conflicts between people leads to the analysis of the equilibrium and disequilibrium regimes between the parties;

5. The combination of the equations of state with the goals of the conflicting sides allows us to investigate the static and dynamic modes of the behavior of human systems under changing conditions;

6. In particular, with the help of various quantitative interpretations of the non-equilibrium regime of mutual pressures, the potential gains and losses of the parties can be assessed, which in turn can serve as a platform for assessing the chances of victory or defeat of the conflicting parties.

References

1. Pavel Barseghyan (2017) "Elements of the Mathematical Theory of Human Systems. Part 1: Assessment of the Results of Human Actions and Activities Based on the Method of State Equations " *PM World Journal Volume 6, Issue 12 December 2017* – 13 pages. http://pmworldjournal.net/wp-content/uploads/2017/12/pmwj65-Dec2017-Barsegyan-mathematical-equation-of-human-systems-part1-featured-paper.pdf

2. Niklas Luhmann (2013). *Introduction to Systems Theory*, Polity.

3. Sociology and the New Systems Theory: Toward a Theoretical Synthesis, by Kenneth D. Bailey

4. The Flaw of Averages: Why We Underestimate Risk in the Face of Uncertainty. 2009, by Sam L. Savage.

5. Pavel Barseghyan (2014) "**Equilibrium and Extreme Principles in Discovering Unknown Relationships from Big Data: Part 2: Non-statistical Mathematical Methods in Project Management**" *PM World Journal Volume 3, Issue 1 January 2014* – 14 pages. http://pmworldjournal.net/wp-content/uploads/2014/01/pmwj18-jan2014-barseghyan-big-data-part2-non-statistical-mathematical-methods-FeaturedPaper.pdf

6. Pavel Barseghyan (2009) "**Human Effort Dynamics and Schedule Risk Analysis**". *PM World Today* – March 2009 (Vol XI, Issue III). 13 pages. http://www.scribd.com/doc/114481457/Human-Effort-Dynamics-and-Schedule-Risk-Analysis

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