Elements of the Mathematical Theory of Human Systems, Part 5: Quantitative analysis of the benefits and losses of two human systems in the regime of confrontational equilibrium of their interrelations

By Pavel Barseghyan, PhD

Abstract

Quantitative analysis of confrontation and acute conflict situations between human systems makes it possible to evaluate and predict the results of different scales of collisions between people in the form of potential benefits and losses.

From the point of view of an adequate quantitative representation of the confrontation between human systems, it is advisable to use such parameters and notions that are universal in nature and invariant with respect to the systems under study.

In other words, these models should operate with universal concepts such as stability and equilibrium of the state of the system, conflict of interests of the opposing sides, the magnitude of the expected benefits, losses and mutual concessions, and so on, which are characteristic of all opposing and conflicting human systems.

Like other activities carried out by people, the process of conflicts between them is also a certain sequence of their mutual actions, which can be quantitatively described by the equations of state of opposing human systems.

This article is devoted to the qualitative and quantitative assessment of losses arising as a result of the conflict between two human systems.

In the first part of the research, the graphic analysis of the process and results of the conflict are carried out, and the second part is devoted to the development of linear and nonlinear mathematical models of confrontation between people and their losses.

Mathematical models of confrontation of two human systems, presented in this article, will later become a platform for quantitative modeling of more complex situations in the clash of three or more conflicting parties.

Key words: Human systems, mathematical theory, state equations, equilibrium, non-equilibrium, benefits, losses, conflicts, confrontational equilibrium, interrelations, pressure on human systems, upper limits of mutual pressure, power of human systems

Introduction

To ensure the safety, prosperity and sustainable development of human systems, a comprehensive quantitative analysis of the equilibrium and non-equilibrium states of their interactions is necessary [1].

The method of equations of state of human systems is a convenient tool for the practical realization of this goal, namely the representation of the relationships between these systems in the form of a system of equations of states, where each of these equations reflects the behavior of the relationships between the two of these systems.

In addition, in modeling, it should be taken into account that the interactions between pairs of human systems are not symmetrical, and for this reason, the mutual influences of the parties are represented by separate equations, each of which splits into two equations, one of which presents a positive pressure on the opposite side, and the other - negative pressure on the same opposite side.

Thus, the mathematical model of the relationship between each pair of interacting human systems in the general case is a system of four equations of state.

This means that with the increase in the number of human systems n, the number of equations of state of their mutual relations will increase sharply and in the case $n = 3$ for this purpose we will have 12 equations, and in the case $n = 4$ we will deal with 24 equations.

In addition, since the pressure on each side of the confrontation can be divided into different types of pressures, such as political pressure, economic pressure, financial pressure, etc., the number of equations representing the state of human relations can be increased even more rapidly.

Naturally, under such conditions, the targeted use of the behavioral models under consideration based on a large number of state equations representing the relationships between human systems is possible only within the framework of modern expert information systems.

On the other hand, in cases where the relationships between human systems are only of a confrontational nature or only the nature of cooperation, the number of equations in the corresponding models can be reduced by half.

From a practical point of view, mathematical modeling of the relationships between the three human systems is of great interest, since this case is relatively simple from the mathematical point of view, but it is already quite complex in the meaningful sense.

In particular, in this case it is already possible to make an object of quantitative consideration the problems of the emergence and degradation of coalitions of people (important from the point of view of the productivity of design teams), organizations (important from the point of view of business) and countries (important from the geopolitical point of view).

In any case, a quantitative analysis of the relationships of an arbitrary number of human systems is based on models of interaction between two human systems that are the subject of further consideration.

Analysis of the confrontation between the two human systems in terms of their potential benefits and losses

Depending on the capacity, scale, resources and psychology of the conflicting parties as well as other circumstances, any pressure on the opposite side may have certain consequences.

If it is a positive pressure (R_+), then it can become a source of L losses for the opposite side, and if it is a negative pressure R_- , which contributes to the activities of the opposite side, can be a source of benefits for him.

The positive pressure of the first of the two conflicting sides on the second will be R_{+12}, and the positive pressure of the second side of the confrontation on the first side will be R_{+21}.

To model the behavior of the conflicting parties, the dependence of the losses of sides on external pressure is of fundamental importance. A qualitative graphic picture of this dependence is shown in Fig.1, where the dependence of the loss L_2 of the second party of the conflict from the pressure of the first party is depicted.

In addition, there is a certain upper limit R_{+UL2} for the current pressure value R_{+12} of the first human system to the second when approaching which the loss of the second side of the confrontation increases sharply.

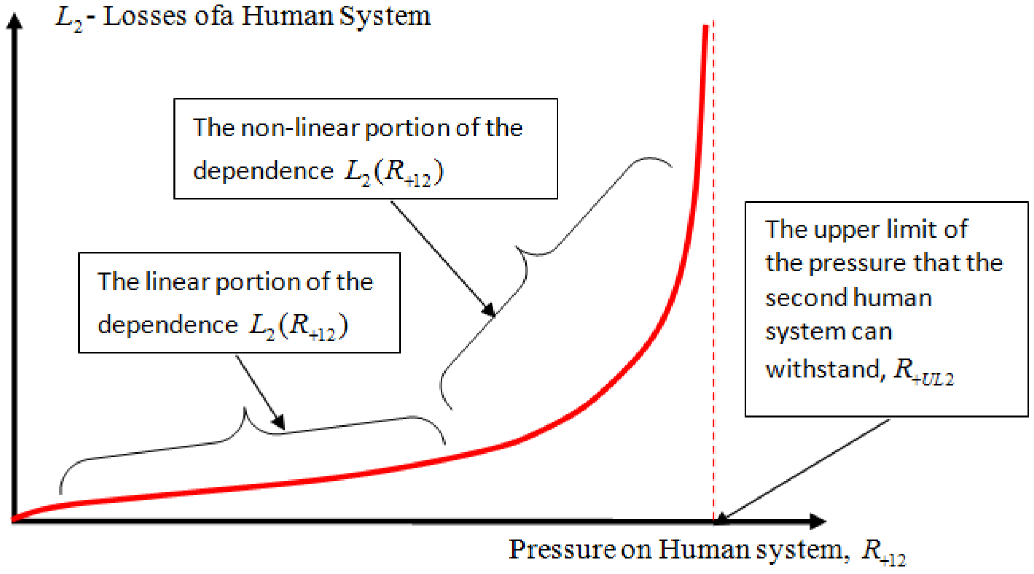

Fig.1. The losses L_2 of the second human system as a result of the pressure R_{+12} of the first system

This value shows the degree of resistance of the second human system to external pressure. The first human system also has exactly the same characteristic, which, together with the characteristic of the second side of the confrontation, is presented in Fig.2.

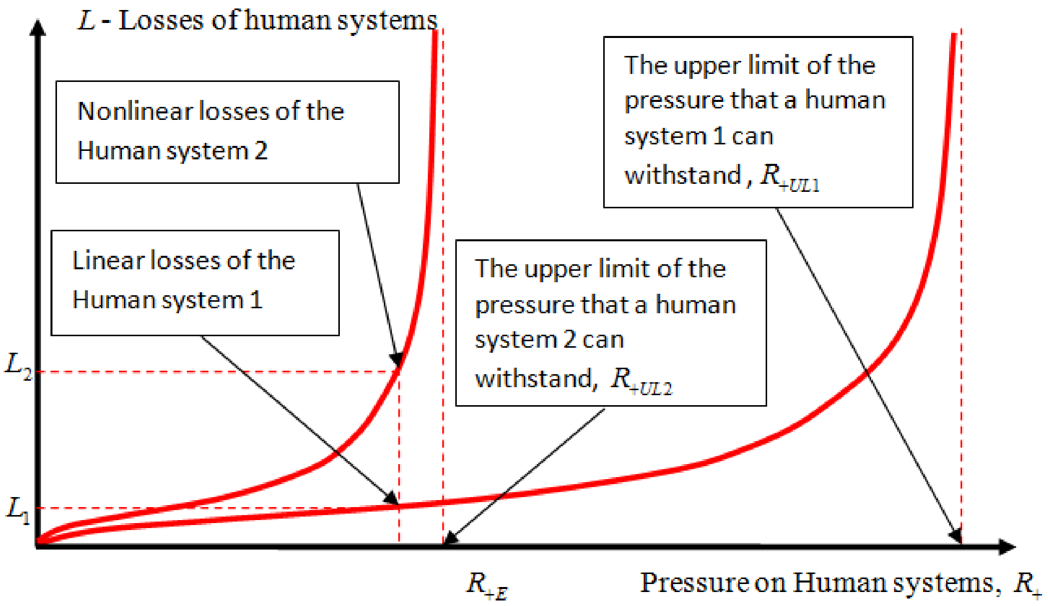

Fig.2. Equilibrium in the confrontation and conflict of two human systems with equal mutual pressure R_{+E}, but different losses L_1 and L_2. The equilibrium is achieved due to the relatively large nonlinear losses L_2 of the weaker human system 2.

These characteristics, as shown in Fig.1, have linear and nonlinear domains that underlie analysis and estimates of the sides' losses.

In particular, the Fig.2 shows a situation where the two sides of the conflict are in equilibrium, exerting equal pressure R_{+E} on each other.

But this equilibrium for the conflicting parties has different prices, which are expressed by different values of their losses.

Thus, a relatively weak second human system is able to achieve equilibrium due to the fact that it operates in a heavier nonlinear mode and has more L_2 losses, and the stronger first human system operates in a relatively easy linear regime and has less L_1 losses.

The impact of changes in the mutual pressures of human systems on their losses and benefits

Consider the behavior of two human systems in confrontational equilibrium, related to possible changes in the parties' priorities and mutual pressure during the settlement of the conflict between them.

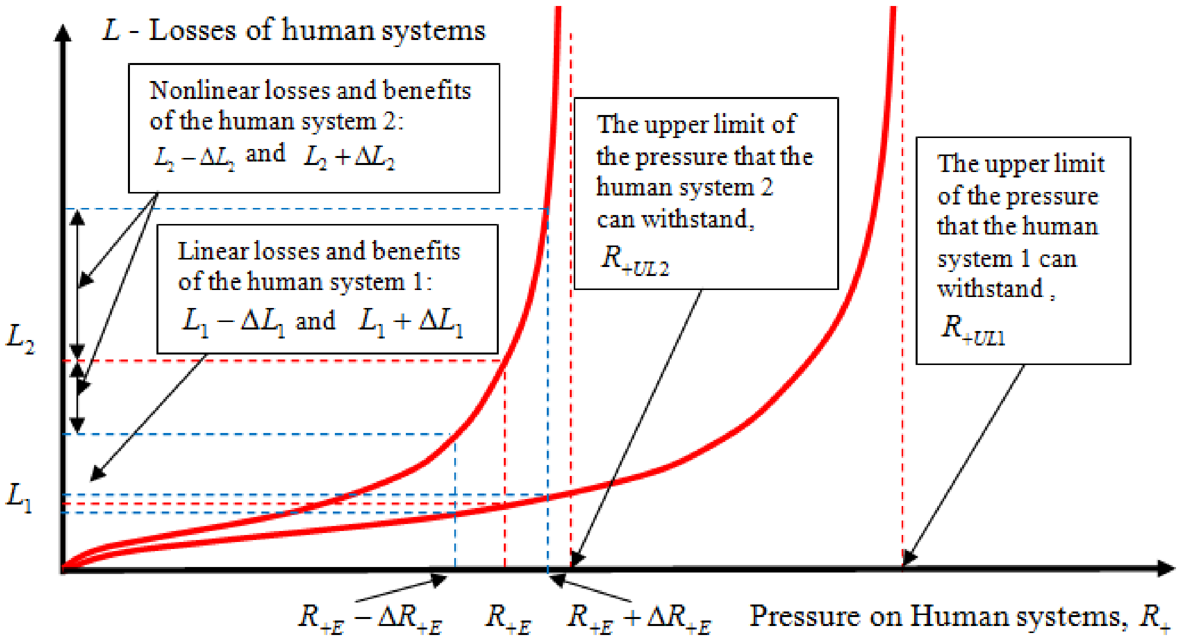

Fig.3 Influence of changes in mutual equilibrium pressures "$-\Delta R_{+E}$" and "$+\Delta R_{+E}$" on linear losses and benefits of the human system 1 "$-\Delta L_1$" and "$+\Delta L_1$" and on nonlinear losses and benefits of the human system 2 "$-\Delta L_2$" and "$+\Delta L_1$"

Having this goal, let's continue our discussion of the situation presented in Fig.2, related to the estimates of the parties' losses and benefits, as a result of the strengthening or weakening of confrontation.

Such a discussion can be continued in two ways, based on deterministic and probabilistic approaches to the problem. The first is simpler in terms of its practical implementation, but has relatively limited scope for reflecting the essence of the conflict.

The probabilistic approach is more complex, but it has much wider possibilities, especially in terms of disclosing and explaining the possible risks associated with the conflict.

First, let's discuss the issue of comparing the benefits and losses of the conflicting parties within the framework of the deterministic approach (Fig. 3).

In order to analyze the behavior of powerful and weak human systems in a conflict, consider changes in mutual pressure between them around the equilibrium point R_{+E}.

According to Fig.3, with the value of the mutual pressure equal to R_{+E}, the average loss of the first human system will be L_1, and the losses of the second human system will be L_2.

If, for any reason, the confrontation between the parties increases by an amount ΔR_{+E}, the system will reach a new equilibrium with the value of the mutual pressure $R_{+E} + \Delta R_{+E}$.

In such a situation, the loss $L_2 + \Delta L_2$ of a weak human system due to the nonlinear nature of the dependence $L_2(R_{+E})$ will be much greater than the loss $L_1 + \Delta L_1$ of a strong human system, which is relatively small due to the linearity of the functional dependence $L_1(R_{+E})$.

This means that for a powerful party pursuing aggressive goals, the strengthening of confrontation is beneficial, since increasing the mutual pressure leads to relatively large losses for the weak human system.

For the same reason, a decrease in the value of the mutual pressure between conflicting parties by an amount ΔR_{+E} reduces the loss of a second or weak human system by an amount ΔL_2 which is greater than the reduction in the losses ΔL_1 of the powerful first human system.

This will mean that reducing tensions between conflicting human systems is always beneficial to the weak side of the conflict, as it brings greater relative benefits to him.

Analysis and assessment of losses of conflicting parties by means of the state equations of relations between human systems

For a quantitative description of the conflict between people, let us consider the equation of state of the human system in its general form [1]

$$N * T * M * P_{Max} = W * D, \qquad (1)$$

where N - the number of people involved in the conflict, T - the duration of the conflict, M - the motivation of people, P_{Max} - the effectiveness of people's activities or simply their productivity, W - the scope or scale of activities related to the conflict, D - the average level of difficulties faced by people associated with conflict .

For the first party of the conflict the equation (1) will have the following form:

$$N_1 * T_1 * M_1 * P_{Max1} = W_1 * D_1. \qquad (2)$$

Considering the amount of activity W_1 as a source of pressure R_{+12} of the first side of the conflict on the second side, we can estimate this pressure in a linear approximation as follows

$$R_{+12} = k_1 * W_1, \text{ or } W_1 = \frac{R_{+12}}{k_1}, \qquad (3)$$

where the coefficient k_1 indicates the rate of pressure accumulation on the second side due to the actions of the first side of conflict.

Substituting the value W_1 from expression (3) into the equation (2), one can obtain a new equation of state for the first human system, which also includes the pressure R_{+12}

$$k_1 * N_1 * T_1 * M_1 * P_{Max1} = R_{+12} * D_1 . \qquad (4)$$

Hence, for the pressure R_{+12} one can obtain

$$R_{+12} = \frac{k_1 * N_1 * T_1 * M_1 * P_{Max1}}{D_1} . \qquad (5)$$

This resulting expression, which is one of the options of the equation of state of the first human system, by means of certain parameters of the equation (5) can be functionally related to parameters that represent the behavior of the second human system.

To establish such a functional connection, one can use parameter D_1 that is a reflection of the difficulties associated with the actions of the first party with respect to the second party of the conflict.

Obviously, the greater the power H_2 of the second human system, the more difficult it will be to put pressure on it, that is, in the linear approximation we will have

$$D_1 = k_{H12} * H_2 , \qquad (6)$$

where the coefficient k_{H12} represents the difficulty or resistance exerted by the unit of power of the second party of the conflict to the first party.

Substituting expression (6) into the equation of state (5), one can obtain

$$R_{+12} = \frac{k_1 * N_1 * T_1 * M_1 * P_{Max1}}{k_{H12} * H_2} , \qquad (7)$$

The resulting relationship is a new equation of state that already reflects the behavior of the unified system of the two conflicting parties, since it contains the parameters of both sides.

Let's continue the analysis of the equation of state (7) using the linear portion of the functional dependence $L_2(R_+)$ shown in Fig. 3, which can be approximated by the following formula

$$L_2 = k_{L12} * R_{+12} , \text{ or } \ R_{+12} = \frac{L_2}{k_{L12}} , \qquad (8)$$

where the coefficient k_{L12} represents the magnitude of the loss of the second party to the conflict caused by unit pressure from the first human system.

Substituting the value R_{+12} from the expression (8) into the equation (7), we obtain another equation of state for the united confrontation system

$$\frac{L_2}{k_{L12}} = \frac{k_1 * N_1 * T_1 * M_1 * P_{Max1}}{k_{H12} * H_2}, \qquad (9)$$

Losses of Human Systems, $L_2(H)$

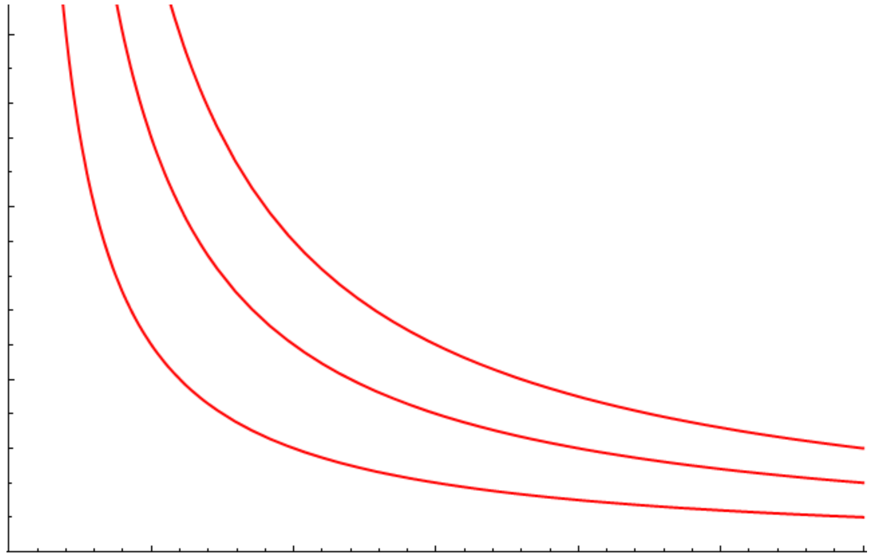

Fig.4 Dependence of loss of the human system on its own power

Hence for the loss L_2 of the second side of conflict one can obtain the following equation of state

$$L_2 = \frac{k_{L12} * k_1 * N_1 * T_1 * M_1 * P_{Max1}}{k_{H12} * H_2}, \qquad (10)$$

which includes many fundamental functional relationships between the system parameters of the conflicting parties.

In particular, it contains a functional relationship between the loss L_2 and power H_2 of the second human system, which has the form of inverse proportionality (Fig. 4).

The same assertion is symmetrically true in the case of the first human system whose behavior can be described by the equation of state, an analog of the equation (10).

Nonlinear deterministic model of an acute conflict

During an acute conflict, the parties in every possible way tend to maximize the pressure on the opposite side, which corresponds to the nonlinear regime shown in Fig.1.

This means that the linear approximation (8) of the functional dependence $L(R_+)$ cannot adequately reflect the essence of the acute conflict between the parties and that in such situations only nonlinear mathematical models can satisfy these requirements.

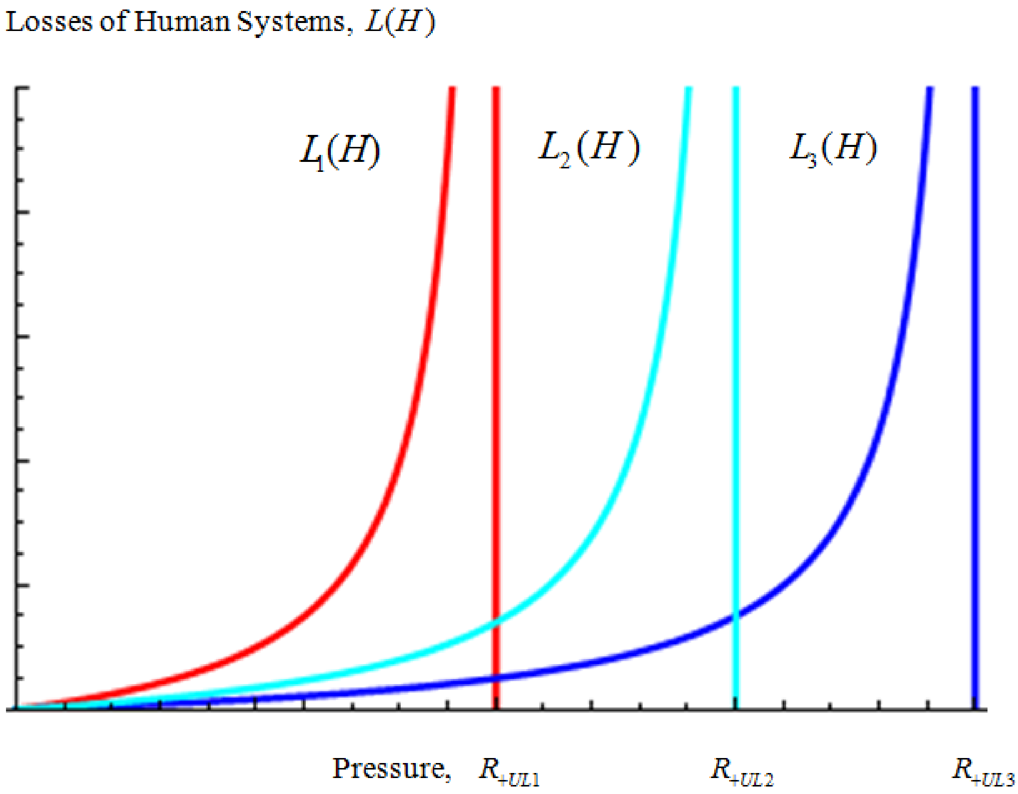

Fig.5 The dependence of losses suffered by human systems on the pressure exerted on them

In particular, the mathematical model of conflict can satisfy the stated requirements, which takes into account the upper limit of the pressure R_{+UL} on the human system, when approaching to which the losses of the system sharply increase.

The simplest mathematical model, corresponding to these requirements, can have the following form:

$$L_2 = \frac{k_{L12} R_{+12}}{R_{+UL} - R_{+12}}, \qquad (11)$$

the graphical view of which is shown in Fig. 5 for three human systems with different upper limits of pressures on them.

Solving expression (11) with respect to the pressure R_{+12}, one can obtain

$$R_{+12} = \frac{L_2 * R_{+UL}}{L_2 + k_{L12}} \ . \qquad (12)$$

Substituting formula (12) into the equation of state (7), one can have:

$$\frac{L_2 * R_{+UL}}{L_2 + k_{L12}} = \frac{k_1 * N_1 * T_1 * M_1 * P_{Max1}}{k_{H12} * H_2}, \qquad (13)$$

Solving this equation with respect to losses L_2, one can obtain:

$$L_2 = \frac{k_{L12} * k_1 * N_1 * T_1 * M_1 * P_{Max1}}{R_{+UL} * k_{H12} * H_2 - k_1 * N_1 * T_1 * M_1 * P_{Max1}}, \qquad (13)$$

which establishes functional links between different parameters of the unified human system, which includes both sides of the conflict.

These functional relationships are crucial for the management of human systems, since such management is usually achieved by purposeful changes in the values of high-level parameters.

These changes, in turn, lead to non-linear and, often, difficult predictable changes in the values of other parameters.

Precisely in order to predict such changes, functional links between the parameters of the unified human system are needed.

The generalized price of mutual losses and the policy of managing the behavior of human systems through mutual intimidations

With the same logic by which the expression (13) of the loss of the second party to the conflict was obtained, one can obtain a symmetrical expression for the loss of the first human system.

$$L_1 = \frac{k_{L21} * k_2 * N_2 * T_2 * M_2 * P_{Max2}}{R_{+UL1} * k_{H21} * H_1 - k_2 * N_2 * T_2 * M_2 * P_{Max2}}: \qquad (14)$$

Besides, the losses of human systems can be of different characters. They can be economic, financial, moral, political, people can lose health, power, life and all these losses have their generalized price.

The gradual increase in losses causes people to be cautious, sober, alert, which with growing loss turns fear, and then into horror, which is associated with the risk of great loss, including the death of people, the failure of organizations and enterprises, and finally, the destruction of countries and civilizations.

In this sense, the most important and essential part of the behavior of human systems associated with conflicts is the policy of mutual threats and fear.

A quantitative analysis of this problem is reduced to describing the nonlinear character of the large losses of human systems by means of the equations of state and taking into account the probabilistic nature of the risks accompanying large-scale collisions of people.

Conclusions

1. The quantitative interpretation of the loss of human systems in conflict situations as a result of the opponent's actions makes it possible to use the method of state equations for estimating and predicting the losses of the sides of confrontation.

2. When approaching the upper limit R_{+UL} of the possibilities of the conflicting parties, the behavior of human systems becomes nonlinear, accompanied by a sharp increase in losses.

3. Comparatively simple linear mathematical models can be used to analyze "mild" conflicts, which are far them of the upper limit R_{+UL}.

4. Nonlinear mathematical models used to analyze acute conflicts and collisions of human systems can serve as a platform for the development of analytical methods for predicting nonlinear risks near the upper limit R_{+UL}.

5. Linear and nonlinear models for estimating the losses associated with conflicts allow us to construct mathematical descriptions of the conflicts between human systems with different capacities, technologies and resources.

6. In addition, linear and nonlinear loss estimation models can serve as a basis for constructing mathematical models of collisions of coalitions and unions of human systems.

Future research

The problems discussed in the article concern the area of deterministic analysis of the behavior of two conflicting human systems and are based on the method of the equations of state.

The obtained results, in particular quantitative estimates of the losses of the conflicting parties on the basis of a single conceptual approach, create the prerequisites for a deeper study of the behavior of human systems.

As for the specific continuations of this work, the following issues are at the forefront:

1. Development of methods for probabilistic analysis of conflicts of human systems that take into account the random nature of the parameters of the equations of state which allow predicting non-linear risks associated with the conflict.

   The point is that the judgments and conclusions made above are valid for the average values of the parameters of human systems, whereas the random variation of the values of the same parameters can contain valuable information for predicting and controlling the behavior of the conflicting parties.

2. Development of deterministic and probabilistic methods of analysis and synthesis of conflicts between three or more human systems.

   The point is that, in the case of a conflict between three and more human systems, using mathematical models of bilateral relations, it is possible to study the mechanisms of formation and destruction of coalitions and unions of human systems.

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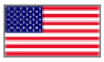

Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia).

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