# Elements of the Mathematical Theory of Human Systems

## Part 6: Human Life as a Project (The differential equation of human life)[1]

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**Abstract**

One of the main trends in modern methods of social management are the attempts to consider social life as a process of implementing large and small projects.

From this point of view, human life and the whole civilization can also be viewed as projects of various scales that have a beginning, a course of life and its end.

If we consider the life of different scale of human systems from the point of view of the dynamics of their abilities and skills, we can easily notice that these abilities grow from the birth of a person or from the appearance of larger human systems, at some age or after some time they reach a maximum. and then follows their decline, aging, and the end of life or activity.

This paper is devoted to mathematical modeling of the entire human life cycle using static and dynamic approaches for this purpose.

**Key words**: mathematical model of human life, differential equation of human life, logistic function, dynamics of human abilities, institutional model of human systems, quantitative modeling of the human aging process.

**Introduction: Human life as a project**

The rapid development of methods and approaches to planning and implementing projects gradually creates a convenient environment for further generalizations of model representations of projects that can be used to promote quantitative methods in other areas of human activity [1].

In particular, we are talking about the fact that over time the number of attempts to consider the life of society as a process of implementing various types of projects increases.

These trends, in turn, create a favorable environment and a clear platform for the further development of methods in project management.

In addition, there are a number of fundamental ideas and approaches through which it is possible within the framework of the same methodology to consider not only the key problems of

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quantitative description of projects, but also many issues related to public life, which still do not have adequate quantitative solutions.

The idea of such a fundamental nature is the process of implementing projects and various manifestations of social life as a process of growth and accumulation, combined with the development of appropriate quantitative methods and models.

In particular, a similar idea was implemented in [2], where the project implementation process is considered as a growth process using logistic differential equations.

Typically, the methodologies used for project management assume that people's performance remains unchanged during project implementation.

This statement is acceptable only for those projects that have a relatively short duration, during which the productivity of people does not change significantly.

But if we are talking about long-term projects, or other long-term actions and activities of people, the statement about the constancy of their performance no longer corresponds to reality, which, in turn, causes incorrect estimates and decisions.

Such a long-term project can be considered the life of a person also, during which his/her abilities, which in a particular case can be identified with his/her productivity in a normal working process, are subject to significant changes with age.

This means that in order to view society's life as a set of parallel projects, it is first necessary to be able to do the same at the human level, since the person is a cell and unit of social life.

Having as a platform a quantitative model of human life from birth to the end of life, one can proceed to more complex cases of the dynamic representation of the coexistence of people and the life of society as a whole.

Since the basis of human activity are the abilities and possibilities of people, then human life should be viewed as a unity of the processes of growth of his abilities, and then their decline after a certain age.

To achieve this goal, namely, to describe the process of human life from birth to its end, it is necessary to have a flexible mathematical model that can adequately describe the slow growth of a person's abilities and skills after birth, their rapid growth during the period of maturity and peaking at a young age and then presenting their decrease due to aging.

The mathematical model of human life must be so flexible that it can also reflect such phenomena as early maturation and late maturation of a person, as well as various rates of aging.

But on the other hand, consideration of these and many other circumstances can lead to insurmountable and unnecessary mathematical difficulties in building a comprehensive quantitative model of the growth and then the decline of human abilities and skills.

Therefore, in the mathematical modeling of such complex phenomena, it is advisable to follow the principle of the gradual complication of the model under development.

This means that at the first stage it is necessary to limit ourselves to building a simple structural model of the phenomenon being studied, provided that the developed model will adequately reflect at least the qualitative aspects of the problem.

Based on these considerations, we consider the dynamics of human capabilities depending on age as a single process, without breaking them into separate components.

**Qualitative and graphical representation of the behavior of the object of modeling: human life as a unity of growth and decline of people's abilities and skills**

In this paper, the object of mathematical modeling is the life of a person and the dynamics of his/her skills and vitality, thanks to which a person ensures his/her normal functioning.

The averaged picture of a person's life process is that every person at his birth has a certain vital force $F(0)$, given to him by genetics, and has a certain growth and development potential, through the realization of which his vital force F increases and reaches its maximum value Fmax at a young age, and then gradually decreases with aging of a person [Fig. 1].

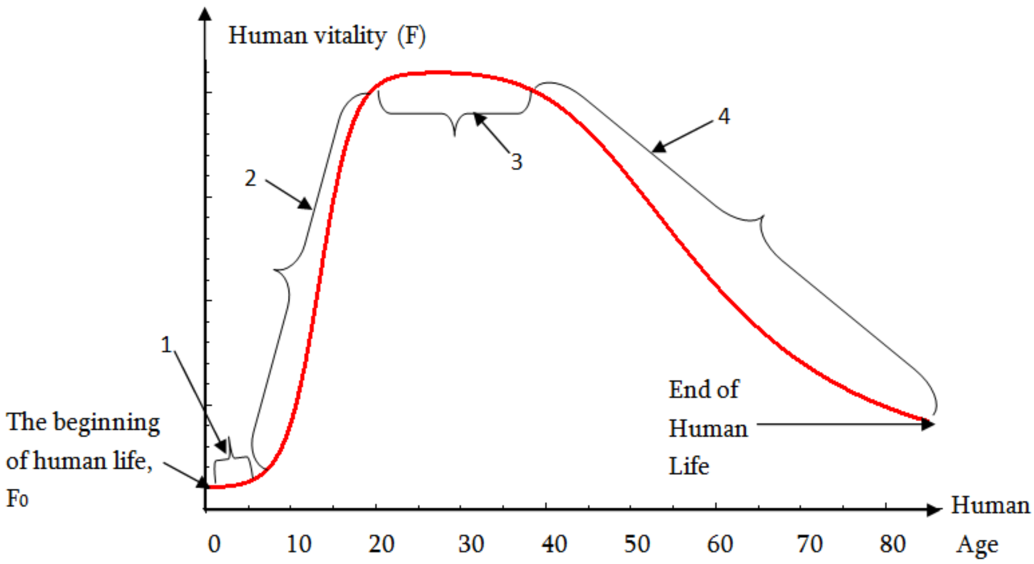


Fig.1: Typical dynamics of human vitality or human life trajectory from birth to maturity and end of life (1 - slow growth of abilities in childhood, 2 - rapid growth of abilities in the process of maturation and formation of a person, 3- the area of comparative constancy of abilities of a person in adulthood, 4 - gradual decrease and decline of human abilities and skills with age and aging)

The image of a person's life process, shown in Fig.1, consists of two qualitatively different parts, namely: the growth of vital forces until reaching the value $F_{max}$ and the fall of these forces until the end of life.

Human vitality or human performance can have various manifestations. In particular, if we consider the issue from the point of view of project management, then this efficiency can be interpreted as human productivity.

Fig.1 shows a picture of a person's average life or, in other words, an idealized image of human life without any serious deviations and cataclysms.

It should be noted here that in the developed mathematical model of human life, as in many other areas, the idea of idealization underlies the quantitative modeling of various phenomena and processes.

Examples of such idealizations in physics are the concept of uniform motion of bodies in mechanics, the idea of an absolutely black body in the theory of radiation and incompressible fluid in hydrodynamics, an ideal gas in thermodynamics, and so on.

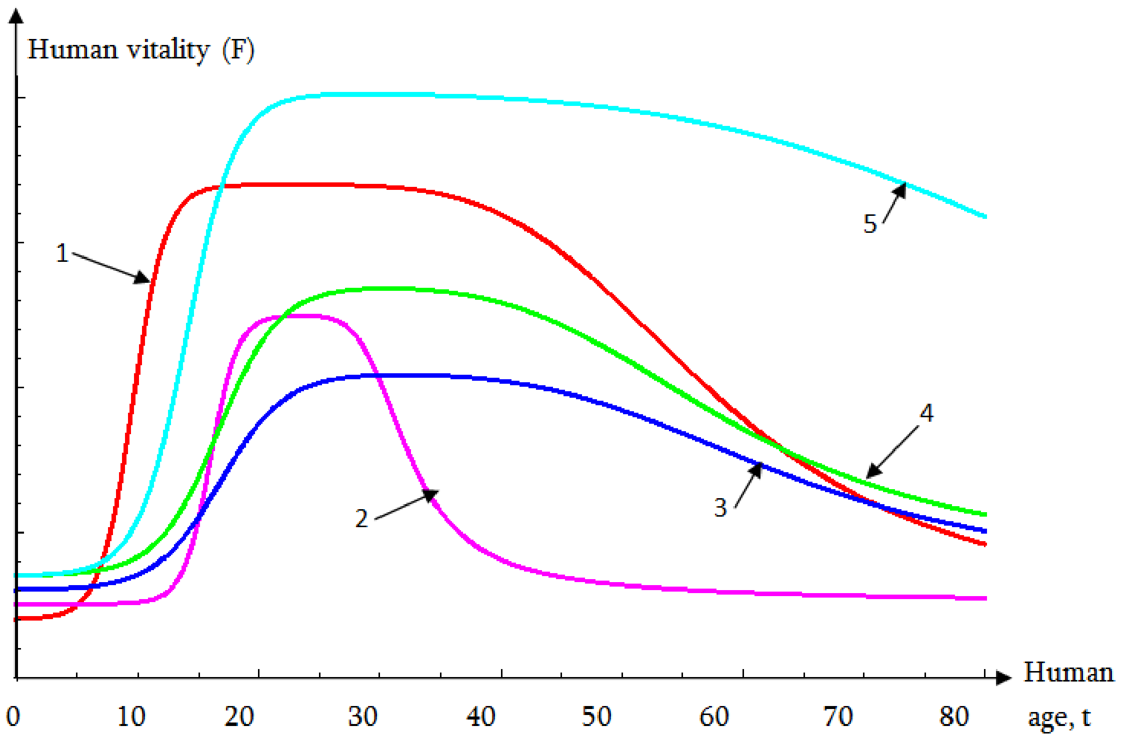


Fig.2: A variety of life trajectories of humans (1 - early matured active person, 2 - late matured person, with low level of abilities and skills and short active life, 3 - person with low level of abilities and skills, 4 - person with medium level of abilities and skills, 5 - early matured person, with a high level of abilities and skills and a long fruitful life)

Similarly, one can talk about idealized human life, ideal connectivity between people and other idealized concepts that can become the core of a quantitative description of people's lives and communication between them [3].

The lives of different people can differ greatly from each other, but there are qualities and characteristics that are the same for the life process of all people, and one of them is that a person’s life is a process of increasing his/her abilities and skills, which reach their maximum value and then decrease with age.

This circumstance is presented in Fig.2, where the images of five different human lives differ from each other, but at the same time they are similar in terms of growth and reduction of a person’s abilities and skills as a function of his/her age.

These similarities and generalities represent the invariant core of human life, on the basis of which one can construct a quantitative description of the averaged human life without any particular upheavals.

As for deviations from the average course of life or small and large shocks that accompany the life cycle of a person, having an average quantitative description of life, it is much easier to describe these deviations as well.

**Static equation of state as an interval mathematical model of human life**

For a quantitative description of the dynamics of people's abilities and skills depending on age, one can use the equation of state of human activities for a finite time interval [3]

$$T * P = W * D, \qquad (1)$$

where $P$ - is the efficiency of human activity, or in the simplest case, the productivity of a person over a certain period of time $T$, parameter $W$ represents the size or scale of human activity, $D$ - is the average level of difficulty of human actions.

Human activity has an average speed $V$, which is defined as the ratio of the scale $W$ of the activity and the duration $T$ of its implementation

$$V = \frac{W}{T}, \quad \text{or} \quad V = \frac{P}{D}. \qquad (2)$$

The essence of the obtained formula (2) is that the course of any activity of a human is similar to overcoming the path of a certain complexity, the speed $V$ of which is directly proportional to the abilities and skills or productivity of human actions $P$ and inversely proportional to the average difficulty $D$ of the path.

Human activity $W$ generates some result $R$ that can be expressed in a linear approximation using the following simple formula:

$$R = k_R W \,, \quad \text{or} \quad W = \frac{R}{k_R} \,, \tag{3}$$

where the coefficient $k_R$ shows a measure of the effectiveness of generating results $R$ by the activities $W$ [4].

Substituting expression (3) into the equation of state (1), one can obtain a new state equation of human activity for the generated result

$$R = \frac{k_R * T * P}{D} \,. \tag{4}$$

From here for the average rate $V_R$ of generation of the results of human activities we get

$$V_R = \frac{k_R * P}{D} \,. \tag{5}$$

If for a relatively short period of time, efficiency together with the speeds of human activity can be considered constant values, and the above expressions (1) and (4) are adequate, then this cannot be asserted for relatively long periods of time, and even more so for the whole course of human life.

In this case, the solution of the problem requires a transition from static descriptions (1) and (4) to a dynamic representation of human activity, which requires more complex mathematical apparatus of differential equations.

**The simplest differential equation of human activity**

In the case when the parameters of human activity depend on time, equation (1) can be adequate only for a small time interval $\Delta t$ during which the volume $\Delta W$ of activity is realized and will have the following form:

$$\Delta t * P = \Delta W * D \,, \quad \text{or} \quad V(\Delta t) = \frac{\Delta W}{\Delta t} = \frac{P(t)}{D(t)} \,: \tag{6}$$

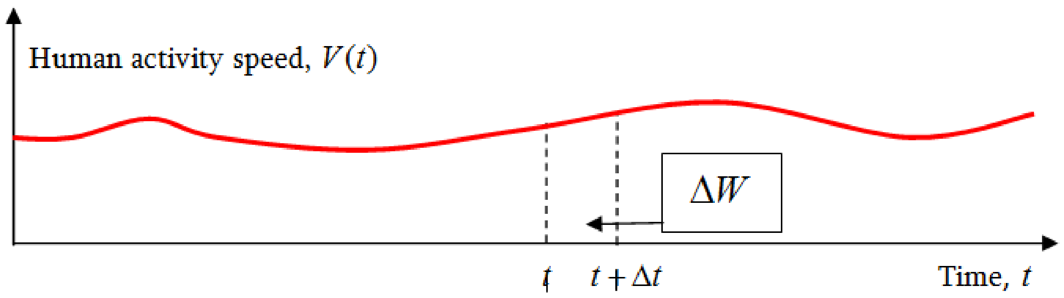


Fig.3 The speed of human activity as a function of time

Passing to the limit $\Delta t \to 0$, from expression (6) we obtain the following differential equation:

$$\frac{dW}{dt} = \frac{P(t)}{D(t)}; \qquad (7)$$

Using the same logic, we can say that during a time interval $\Delta t$ a person produces results $\Delta R$ that can have various manifestations, from the production of a real product to emotional manifestations and pressure on the people around him.

In the same way, using expression (4), one can obtain the differential equation for the function $R(t)$.

$$\frac{dR}{dt} = \frac{k_R(t) * P(t)}{D(t)} ; \qquad (8)$$

Differential equations of state (7) and (8) can be used in different fields of human activity, including the management of various types of projects.

But when using these equations for any particular purpose, it is necessary to have the form of functions $k_R(t)$, $P(t)$ and $D(t)$, which reflect the human qualities and properties of the environment in which the person acts.

Along with other problems, the obtained equations are also suitable for a quantitative description of human life, but for this, first of all, it is necessary to know the dynamics of a person's abilities and skills $P(t)$ as a function of his/her age. To do this, consider the behavior of this function in more detail.

**Quantitative description of human vitality and abilities**

Human vitality is manifested through his/her skills and abilities, which can be represented in various quantitative ways.

Consider one of the possible ways to quantify the dynamics of human skills, based on a hypothesis that allows us to model the process of growth of abilities and skills of a person.

First, we consider the process of the growth of a person's abilities, meaning by this the ideal dynamics of the accumulation of his/her knowledge and skills over the course of a lifetime, the graphic image of which is shown in Fig.4.

The picture shows that a person at birth has certain functional capabilities or abilities $P(0)$, and at an arbitrary age $t$, changing the latter by an amount $\Delta t$ leads to an increase in his/her abilities by an amount $\Delta P = P(t + \Delta t) - P(t)$. In addition, each person has a certain potential $P_{Max}$ for the growth of abilities, which is the upper limit of his/her physical and cognitive development.

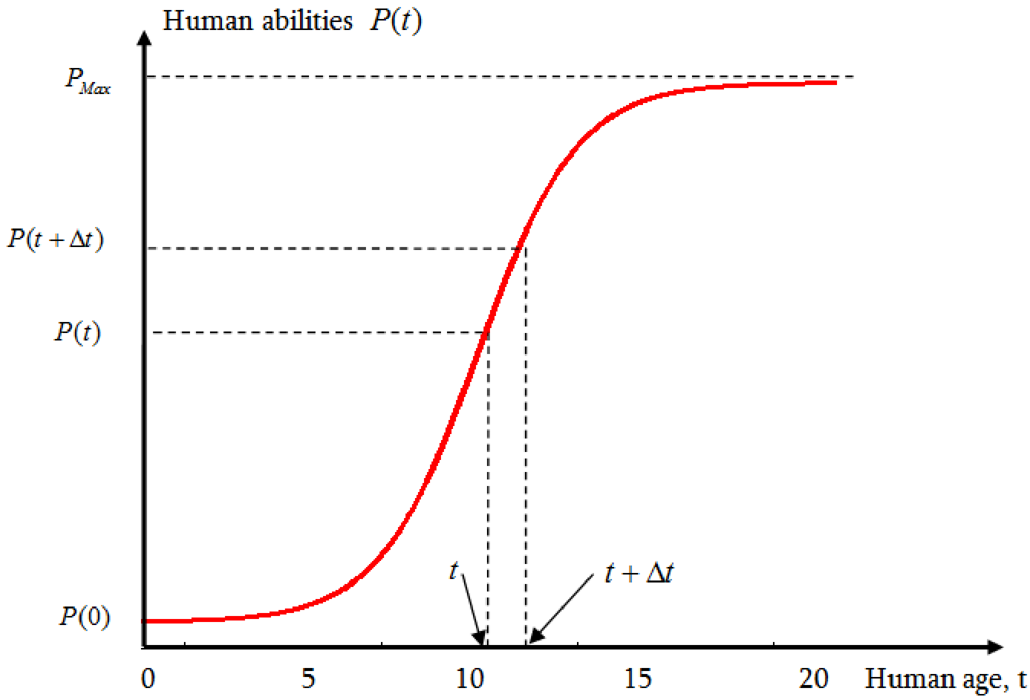


Fig. 4 An idealized picture of the growth of human abilities and skills from birth to young age. The figure does not show the age-related decline in human abilities and skills

Despite the fact that the value $P_{Max}$ is variable and, in particular, depends on the living conditions of humans and the level of development of society, to simplify the process of building a model of net growth of human skills, it is considered a constant value.

**Differential equation of the growth of human abilities**

The derivation of the differential equation for the growth of a person's abilities and skills is based on the hypothesis that the growth $\Delta P$ of human abilities in an arbitrary age $t$ is directly proportional to both the change $\Delta t$ in age and the abilities $P(t)$ already accumulated and the unused resource $P_{Max} - P(t)$.

The mathematical expression of this statement will be as follows:

$$\Delta P = k_G P(t) * [P_{Max} - P(t)]\Delta t, \quad (9)$$

where $k_G$ - is the proportionality coefficient depending on the human's personality characteristics, the level of development of the society and other circumstances.

By dividing the two parts of expression (9) by $\Delta t$ and passing to the limit $\Delta t \to 0$, we get the following differential equation

$$\frac{dP}{dt} = k_G P(t) * [P_{Max} - P(t)]. \tag{10}$$

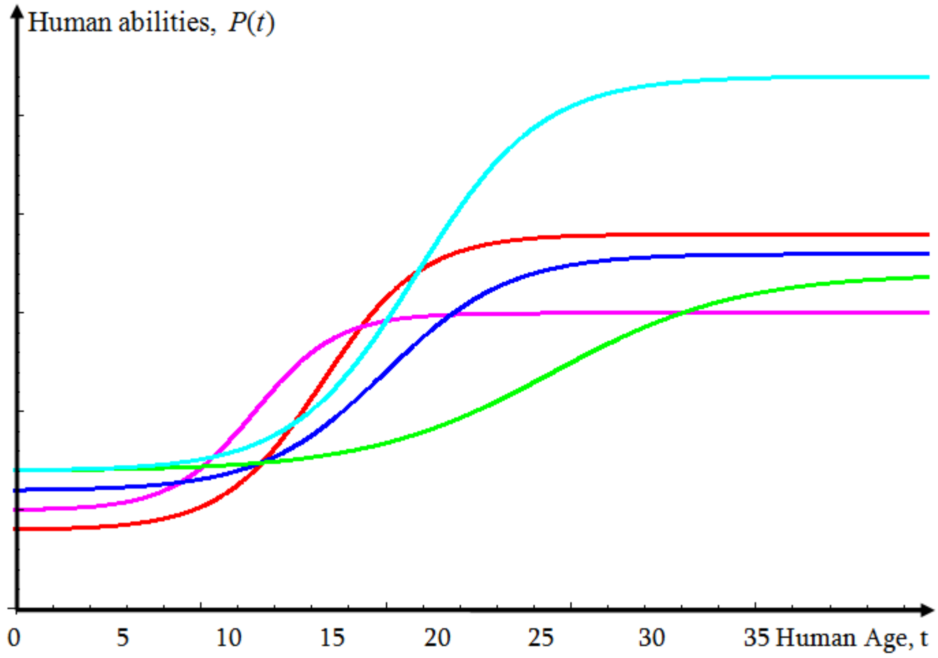


Fig.5. The processes of growth of abilities and skills of people in different conditions without taking into account the effects associated with human aging

This equation is known in mathematics as a logistic differential equation, first published in 1838 by the Belgian mathematician Verhulst and having a wide range of applications in biology and demography [5].

The solution of the equation (10) is the logistic function, which, taking into account the initial condition $P(0)$, has the following form:

$$P(t) = \frac{P(0)P_{Max}}{P(0) + (P_{Max} - P(0))Exp(-k_G P_{Max} t)}, \tag{11}$$

which is graphically shown in Fig.5 for the growth of abilities and skills of five different people.

The differential equation of the growth of human abilities and skills (10) and its solution (11) in the form of a logistic function describe the process of growth of his/her abilities from birth to the peak of physical and cognitive capabilities.

To complete the quantitative description of the dynamics of skills and abilities as a function of a person's age, it is necessary to expand the capabilities of the constructed growth model in the form of a logistic differential equation in such a way as to cover the second part of human life, where there is a gradual decrease and decline of human vitality.

There are a number of possible approaches to achieving this goal, the main of which is to control the value $P_{Max}$ parallel to the aging of the individual.

For this purpose, both differential equation (10) and logistic function (11) can be used.

**Differential equation of human life**

To achieve this goal using the logistic differential equation, two factors need to be taken into account: first, the decrease in a person's abilities and skills level with age must be described using a function that can later fit the experimental data flexibly and effectively, and secondly, the decline of person's abilities and skills should start after a certain age.

The simplest way to implement such an idea is to use a hyperbolic model of reducing the value of $P_{Max}$, while taking into account that the reduction of a person's vital forces begins only after a certain age $T_0$.

This means that in differential equation (10), the quantity $P_{Max}$ should be replaced by its dynamic variant $P_{DMax}$,

$$\frac{dP}{dt} = k_G P(t) * [P_{DMax} - P(t)], \qquad (12)$$

where the value $P_{DMax}$ is determined by the following expression

$$P_{DMax} = \frac{P_{Max}}{\alpha + \beta t^{\mu}}, \qquad (13)$$

In expression (13), the quantities $\alpha$, $\beta$ and $\mu$ are the parameters controlling the flexibility of the hyperbolic approximation model.

Considering the age delay $T_0$ of a gradual decrease and decline of a person's abilities and skills with age, instead of expression (13) we will have

$$P_{DMax} = \frac{P_{Max}}{\alpha + \beta (t - T_0)^{\mu}}: \qquad (14)$$

Substituting expression (14) into equation (12), one can obtain the final form of the differential equation representing the course of human life and the dynamics of his/her abilities and skills

$$\frac{dP}{dt} = k_G P(t) * [\frac{P_{Max}}{\alpha + \beta (t - T_0)^{\mu}} - P(t)]: \qquad (15)$$

Graphic images of solutions of these equations are shown in Fig.1 and Fig.2.

**Quantitative representation of human life as a system of two differential equations**

Combining equation (15), which describes the dynamics of a person's skills and abilities with equation (8), which in turn describes the dynamics of generating the results of his/her activities, one can obtain the dynamic model of human life in the form of two differential equations

$$\left.\begin{aligned} \frac{dP}{dt} &= k_G P(t) * [\frac{P_{Max}}{\alpha + \beta(t - T_0)^{\mu}} - P(t)], \\ \frac{dR}{dt} &= \frac{k_R(t) * P(t)}{D(t)} \end{aligned}\right\} \qquad (16)$$

These equations describe the core of human activity in the form of a chain of human skills and abilities $\rightarrow$ the possibilities of carrying out its activities $\rightarrow$ the results of these activities. The meaning of this chain is that a person possessing variable abilities and skills $P(t)$, which, due to the properties of the social environment $k_R(t)$, create the possibilities $k_R(t)P(t)$ by which a person overcomes life's difficulties $D(t)$ and creates $R(t)$ results with variable speed $V(t) = \frac{dR}{dt}$.

At the same time, depending on the circumstances, the function $k_R(t)$ that represents the institutional peculiarities and features of the social environment for the abilities and skills $P(t)$ of a person can play both an amplifier role or an attenuator role.

**Conclusions**

1. The mathematical model of human life, presented in the form of a differential equation, creates a wide range of prerequisites for quantitative studies of social life;
2. Studies show that generalizations of static equations of state representing a certain time period of a person's life can serve as a basis for a quantitative description of the whole course of human life;
3. Human abilities and skills $P(t)$, multiplied by the opportunities $k_R(t)$ provided by the public, can generate results $R(t)$ for the progress of society;
4. In the process of creating a public result $R(t)$, a quantity $k_R(t)$ that is a reflection of the institutional structure of society can act as an amplifier of people's skills and abilities, as well as their attenuator;
5. The well-known logistic differential equation can serve as a mathematical model for describing a person's life and the growth of his abilities and skills;
6. The same logistic differential equation, after some modification, can serve as a united model of human life, reflecting both the process of growth of people's abilities and skills at the beginning of their life, and their reduction in the second part of life.

**Continuation of work**

The results obtained in this work are applicable to the processes of nucleation, development and formation of various human systems, as well as for the quantitative description of their life cycles.

The mathematical models developed in this paper are also applicable to a more in-depth quantitative description of human life cycle. In particular, further research in this direction is related to the fact that for a more adequate dynamic representation of human life it is necessary to deepen the dynamic model (16), by dividing it into two interrelated mathematical models, one of which will represent the dynamics of a person's physical abilities and skills, and the second one - the dynamics of his/her cognitive abilities and skills.

Evaluation of the results of human actions implies that in addition to describing the dynamics of a person's skills and abilities, it is also necessary to describe the dynamics of the capabilities of the environment in which a person or a human system operates.

Further steps in this direction are associated with a quantitative description of the environment of human activity, the goal of which could ideally be a quantitative description of the institutional structure of society and, as a result, an assessment of the institutional effectiveness of the human system under consideration.

Another direction of further research is closely related to the fact that the considerations underlying the quantitative model of the human life cycle are applicable to a quantitative representation of the development and dynamics of the civilization level.

Here we are talking about the fact that, like a person's life cycle, civilization also has its own life cycle, its origin, development, reaching a peak, and then falling.

Another direction of future research is that organizations also have their own life cycle, an understanding of the details of which and an adequate presentation of the dynamics of an organization's life can have a decisive influence on the organization of its business.

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 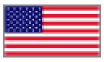

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