Mathematical models of human cooperation and the transformation of human capital into social capital

Part 1. The individual energy field of human interaction with social environment and the formation of social solidarity (asabiya) and social capital[1]

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Abstract

Different societies, along with other features, differ from each other in the degree of cooperation between people, mutual understanding and their social solidarity.

As early as the 14th century, Ibn Khaldun described this group of features of human society with the term asabiya, which indicates the degree of social solidarity and the ability of people to organize coordinated and successful joint activities.

These qualities of groups of people of different size and scale, in turn, are directly related to such a concept, reflecting the internal forces of society, as social capital.

The above features also largely determine the cohesion of society around the ideas of stability and development of countries.

In such conditions, the understanding and control of the internal dynamics of the development of society require a comprehensive study of the problems associated with it, along with other approaches, using the methods of mathematical modeling.

The first part of this work is devoted to the development of mathematical models of interaction between people and their internal harmony (asabiya) at the level of an individual from an energy point of view, meaning the interrelationship and assessment of energy consumed by a person for his/her own vital needs and for the vital needs associated with interactions with other people.

In the second part of the study, mathematical models of social cohesion or asabiya and the social capital of society will be considered, depending on the relevant characteristics of the people that make up society.

Key words: Collaboration, social solidarity, mathematical model, human capital, social capital, asabiya, institutional structure of society

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Introduction: Collaboration of people and their solidarity as the cornerstones of sustainable social life

Even the most simple and superficial observation of people's lives can clearly show that cooperation and solidarity among members of the community are the cornerstone of rational organization of life.

Collaboration between people in combination with solidarity allows them to combine their strength and capabilities to achieve different goals.

Naturally, the main purpose of cooperation between people is to ensure the continuity of life, primarily in the form of ensuring their safety and proper nutrition.

At the same time, the more organized cooperation between people, the higher the likelihood of achieving a specific goal and the less likelihood of risks and losses.

The effectiveness of cooperation is directly related to the physical and psychological features and characteristics of members of society, and their origin goes deep into history.

In this regard, one of the factors directly affecting the effectiveness of cooperation is the attitude of people to the ratio of their personal and social interests.

In other words, this problem is directly related to the attitude of society towards people's self-interest and their willingness to cooperate with others.

On the other hand, it is also clear that different societies, whose psychological picture was created in favorable or unfavorable conditions, treat this key problem differently.

It is also obvious that in parallel with the development of society and the improvement of the quality of life, a person needs less cooperation with other people to ensure basic vital functions.

This circumstance contributes to the alienation of people from each other and, naturally, strengthens egoistic tendencies in society.

All this indicates that a deep study of the phenomenon of cooperation between people is very important from the point of view of ensuring stability and efficiency of society and other human systems of an arbitrary scale.

These studies can be both traditional or qualitative and quantitative, based on mathematical models of egoism and cooperation of people, the study of which is the subject of the present work.

These problems are typical macrosociological problems, and one of the goals of this work is to show that many high or systemic level characteristics of human systems of arbitrary scale directly depend on the specific characteristics and features of people.

Possible approaches to the problem of quantifying self-interest and cooperation of people

Within the framework of the mathematical theory of human systems, where human life is presented as a stream of actions, it is possible to construct various quantitative models of selfishness or self-interest and cooperation of people.

The flows of human actions, as is done in the quantitative description of flows in various fields of science, can be represented using both deterministic and probabilistic mathematical models.

The relative values of self-interest and cooperation in human behavior within the framework of the simplest deterministic approach can be measured by the number of human egoistic actions and actions for cooperation per unit of time (one day, one week, one month, etc.).

These values will correspond to the intensities of human actions of a egoistic nature and actions related to human cooperation.

Looking at the actions of people related to their cooperation in more detail, we see that these actions are heterogeneous, which means that people cooperate more intensively with "close" neighbors than with relatively "far" neighbors.

With a more precise quantitative formulation, one can say that for any person or human system there is a certain degree of connectivity with other people, which is stronger for people who are "close" and gradually weakens with increasing relative distance between people.

A quantitative description of this phenomenon is presented in [1, 2], where connectivity functions are used to estimate and predict system level parameters in the field of electronics [1] and human systems [2].

The connectivity functions used in these works can also quantitatively describe self-interest and cooperation of people, using such concepts as the degree of closeness of people or the relative distance between them in the corresponding multidimensional spaces.

A single function for the quantitative representation of human self-interest and his/her cooperation with other people

Here we are talking about the fact that the principle of absolute egoism in human society and in the animal world cannot fully explain the behavior of either people or animals.

The fact is that the basis of the continuity of life of people and animals is the concept of mutual assistance and cooperation.

In other words, in order to ensure continuity of life, besides taking care of your own safety and nutrition, it is also important to take care of the safety and nutrition of others to some extent.

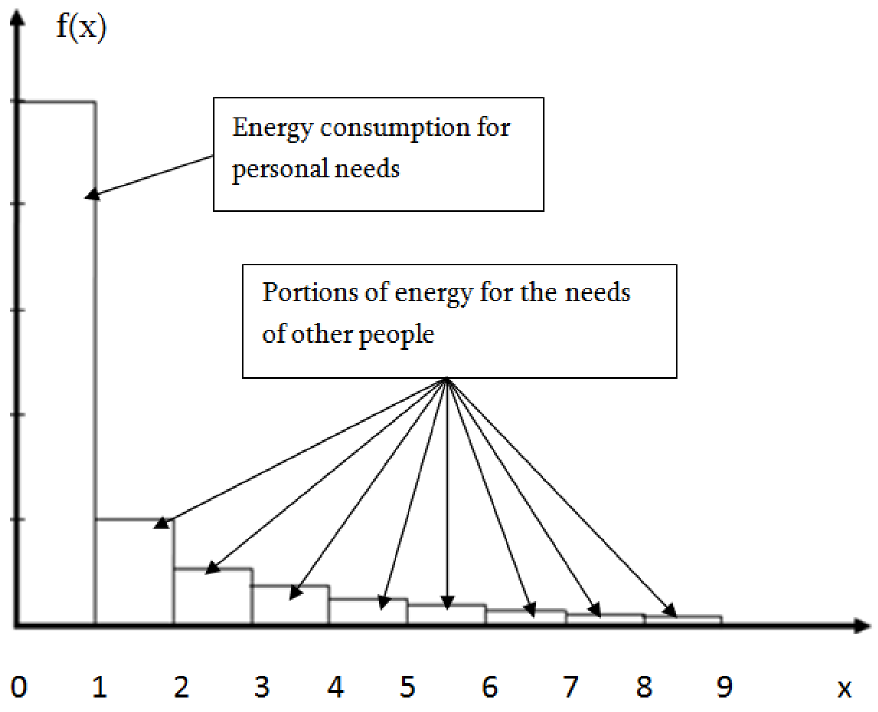
If we approach the question from a purely energetic point of view, one can say that most of the energy received in the form of food is used for its own needs, and the rest or small part of it is used for the needs of others.

At the same time, this small part of the energy is clearly distributed among others, based on the principle of ensuring the continuity of life with the maximum probability.

This means that most of the energy consumed by others goes to the person who has the greatest possible contribution to ensuring the success and continuity of life.

The next largest piece of energy goes to that of the rest, which has the relatively most important role in terms of continuity of life and general well-being, and so on.

In other words, this leads to the fact that the energy that is consumed by an arbitrary person for others has a distribution that meets the requirement of continuity of life and is based on the principle of commonality of people's vital interests.



Fig,1 The distribution of energy consumed by a person for his/her own needs and for the needs of others. Here, the x - axis shows the sequence of people by their contributions in the common well-being, and the rectangular surfaces s_i ($i = 1,2,...,9$) show the relative sizes of the energies spent on a person's neighbors and, therefore, $\sum_{i=1}^{9} s_i = 1$.

The discrete function of such a distribution is graphically illustrated in Fig.1, which shows both the part of human energy consumed for personal needs, and the parts of energy consumed by man for the needs of others.

If, for example, a person spends energy U per unit of time, then the product $s_i U \quad (i = 2,...,9)$ shows how much energy a person spends for the needs of his i -th neighbor.

In addition, the expenditure of each energy unit for cooperation of people leads to a certain result, depending on the aggregate capabilities H of each person, and which, in turn, depends on the experience, knowledge, skills and living conditions of people.

This means that a portion of energy $s_i U$ can generate a result R_i, which is determined by the following formula:

$$R_i = k_{Inst} H U s_i \quad , \tag{1}$$

where the coefficient k_{Inst} shows the degree of effectiveness of the institutional environment in society.

If you leave aside the financial aspect of the problem, the meaning of this simple formula is that to achieve success in any cooperation, one must have an institutional environment, business capacities of people (experience, knowledge, skills) and consume energy for that.

Mathematical model of social solidarity (asabiya) and social capital

Cooperation between people is possible and can give positive results only if their activity is carried out in conditions of social solidarity or asabiya.

The term asabiya was used by Mohammed ibn Khaldun (1332-1406) to describe social solidarity and harmony among people and their civilizational activity, as well as their ability to pursue common goals [4, 5].

From the point of view of civic activity of people, the term asabiya has a semantic similarity with the Gumilev's term passionarity, because, like Ibn Khaldun, he believes that the passionarity of people is more clearly manifested in the period of the birth of civilizations, after which it gradually weakens [6, 7].

Social solidarity and social capital, which by their meaning are in many respects equivalent to Ibn Khaldoun's asabiya and Gumilev's passionarity, along with a verbal presentation can also be described by mathematical models and, thus, reflect important group quantitative characteristics of people.

A person's business or intellectual potential H can also be interpreted as human capital C_H, which, thanks to people's energy U and the effectiveness of the institutional environment k_{Inst}, is transformed into a result of cooperation R in the form of social solidarity (asabiya) C_S or social capital C_c.

From a quantitative point of view, this means that there is a chain of functional dependencies, according to which the intellectual potential H generates human capital C_H, which, in turn, creates social solidarity C_S, and which, in turn, generates social capital C_c.

This chain of functional dependencies, which links the system parameters of social life with each other, is mathematically represented as a sequence of interrelated equations of state that reflect the equilibrium of human life.

Mathematical model of energy distribution for human cooperation

Group characteristics of people can be quantitatively described using the function $f(x)$ shown in Fig.1, but since this function is discrete, it is advisable to replace it with an equivalent continuous function $u(x)$ adequately reflecting the energy flows associated with human interactions for the purposes of subsequent analytical calculations.

For a quantitative analysis of cooperation between people, let's consider a simplified model of society, shown in Fig.2, according to which interacting people are evenly distributed along with the x axis.

If we continue to consider the interaction of people from an energetic point of view, then the function $u(x)$ shown in Fig.2 will represent the energy expended by the person who is at the point 0 of cooperation with people located on the right of the point x.

Consider the behavior of the function $u(x)$ in the interval dx, given that the consumption or absorption of energy along the x axis occurs with a certain intensity $\lambda(x)$, which shows the specified energy consumption per unit length of the same x axis.

In cooperation with people in the interval dx, a person spends energy $u(x) - u(x+dx) = du$, which can also be determined using the function $\lambda(x)$ as follows: $du = u(x)\lambda(x)dx$.

Equating two values of the energy flow du in the interval dx, one can get:

$$u(x)-u(x+dx)=u(x)\lambda(x)dx \text{ or } u(x+dx)-u(x)=-u(x)\lambda(x)dx \quad (2)$$

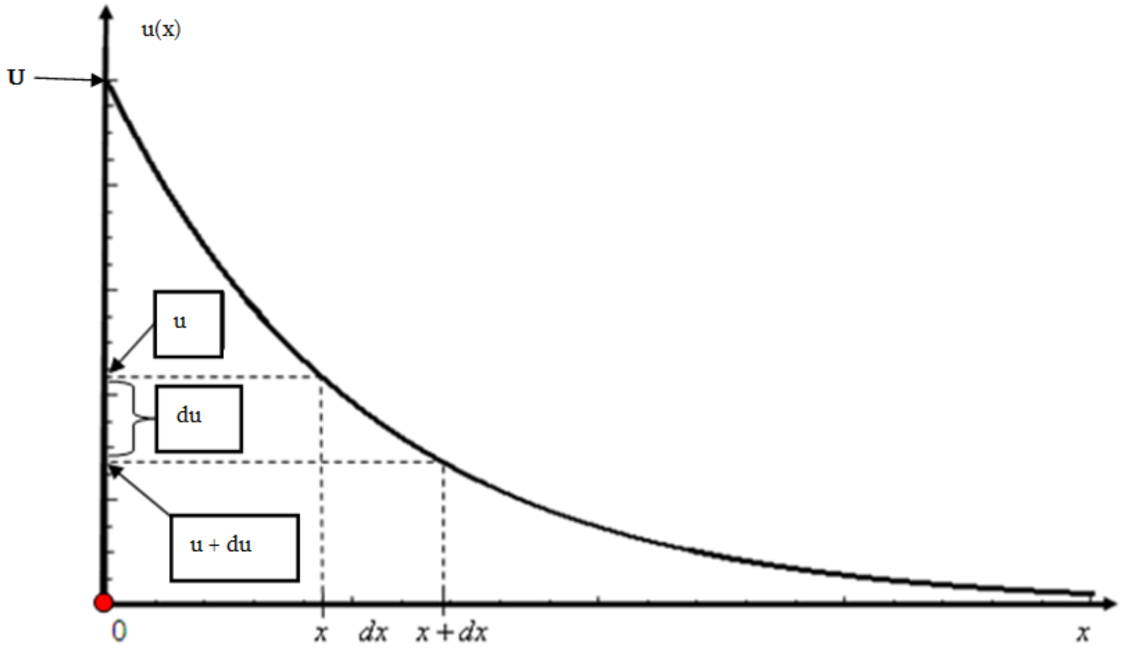


Fig.2 A homogeneous model of society in which people are evenly distributed along the x axis and they spend energy for the purpose of cooperation. The function $u(x)$ represents the energy flow of a person located at point 0 and indicates that people spend more energy on interacting with their immediate neighbors than with those who are far away from them.

The last expression is equivalent to the following differential equation:

$$\frac{u(x+dx)-u(x)}{dx}=-u(x)\lambda(x) \text{ or } \frac{du}{dx}=-u(x)\lambda(x) \quad (3)$$

The solution of this equation for the boundary condition $u(0)=U$ is as follows

$$u(x)=UExp[-\int_0^x \lambda(x)dx], \quad (4)$$

which in the simple case, when, $\lambda(x)=\lambda=Const$, turns into an exponential law of cooperation between man and the environment.

$$u(x)=UExp[-\lambda x] \; . \quad (5)$$

According to this law, the distribution of human energy U, directed to the cooperation with other people, depends on the value of intensity λ or, in general, the behavior of the function $\lambda(x)$.

In the case of large λ intensity values, a person actively interacts only with his/her narrow circle, and his activity is isolated, but in the case of relatively low intensity λ, a person also communicates with those who are relatively far from him.

In addition, Fig.3 also shows that an increase in intensity λ increases the energy expended by a person on selfish goals, while its decrease leads to an increase in people's energy consumption on altruistic goals.

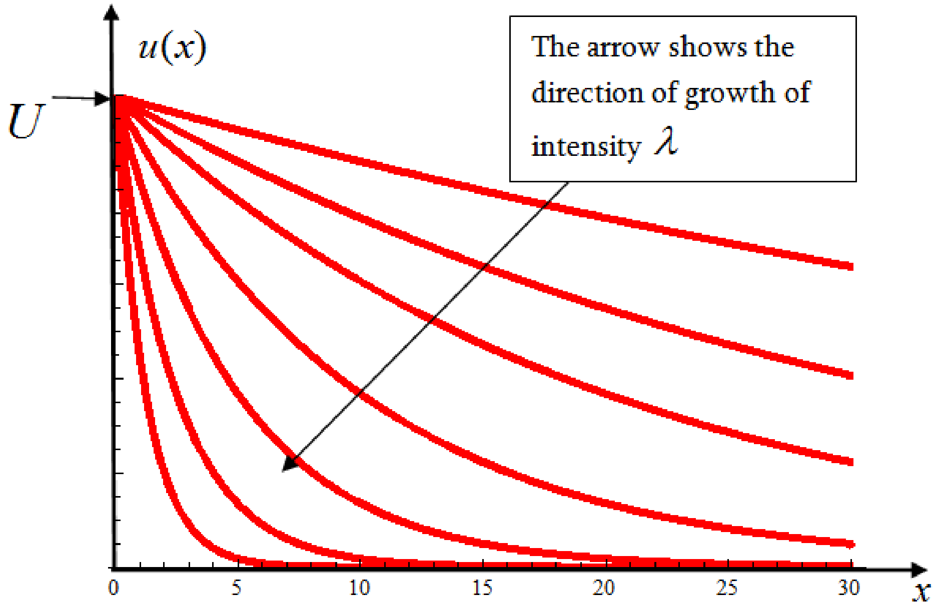


Fig.3 The lower the intensity λ of the flow of energy between people, aimed at cooperation and solidarity between them, the further the energy can spread in the direction of the x axis

Properties of the energy function of human cooperation $u(x)$

To quantify the group characteristics of people, including their cooperation and solidarity or asabiya, social capital and other high-level parameters, it is necessary to know the properties of the function $u(x)$ underlying all of these assessments.

To do this, let's consider the figure shown in Fig.4.

Thus, the value $U - u(1)$ is the energy expenditure of a person for his own needs, which is equal to the value $s_1 U$, and the difference $u(1) - u(2)$ is the energy expenditure of the person for cooperation with the nearest neighbor, equal to s_2U and, similarly, the energy expenditure associated with cooperation with all neighbors can be determined.

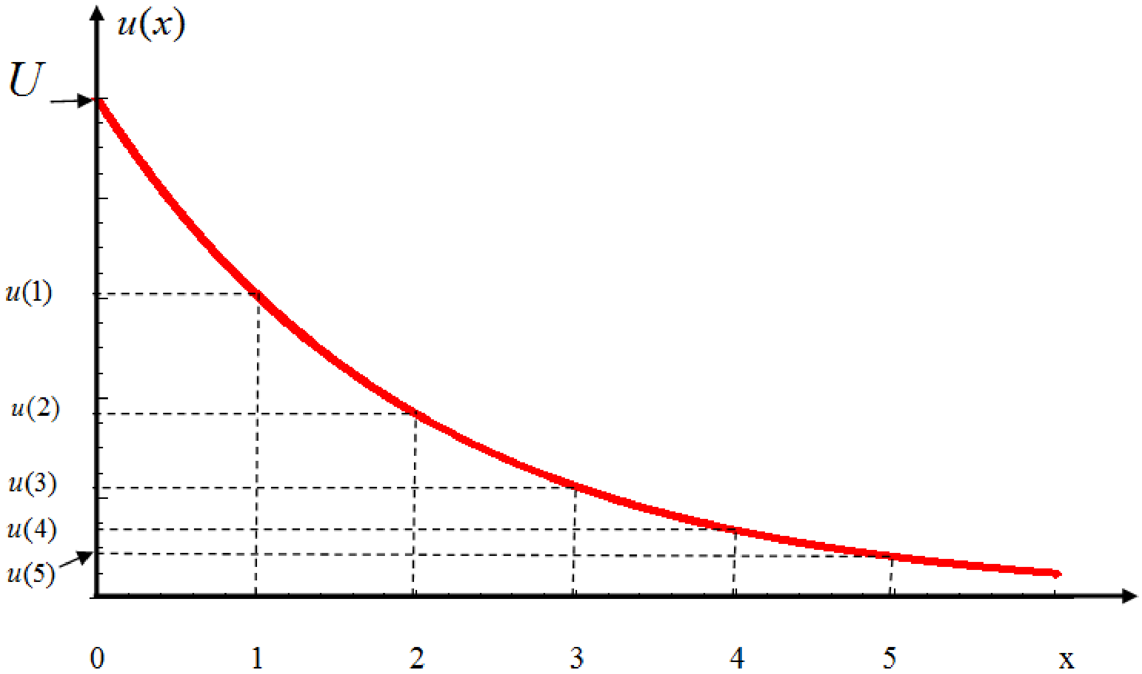


Fig.4 Energy consumption by a person at the origin of coordinates for cooperation with other people, which are evenly distributed along the x axis

The function presented in this figure, which can be called the function of the energetic connectivity of a person with others, shows the distribution of energy consumed by this person to meet his/her own needs and the needs of other people located along the x axis.

If we approach the issue from the point of view of cooperation of a group of N people, in which a person at point 0 also participates, the value $u(N-1)$ will indicate the energy consumption of the latter for cooperation with people who are located on the x axis to the right of the point $N-1$.

And inside the group of N people, the energy consumption of the person located at point 0 for cooperation with other $N-1$ people will be $u(1)-u(N-1)$.

This will mean that, according to expression (1), a person's energy expenditure $u(1)-u(N-1)$ can serve as a basis for estimating various social parameters.

Thus, within the framework of the simple linear approach, thanks to the cooperation between a person at point 0 and others, the generated amount of human capital will be

$$C_H = k_H R = k_H k_{Inst} H[u(1) - u(N-1)] \,, \tag{6}$$

where k_H - is a linear coefficient of human capital C_H generation by using human potential H.

In turn, human capital C_H generates social solidarity or asabiya C_S

$$C_S = k_S C_H = k_S k_H k_{Inst} H[u(1) - u(N-1)], \tag{7}$$

where k_S - is the coefficient of linear approximation of the function $C_S = \varphi(C_H)$.

In the same way, it is possible to represent the generation of social capital C_C through social solidarity or asabiya C_S.

$$C_C = k_C C_S = k_C k_S k_H k_{Inst} H[u(1) - u(N-1)] \tag{8}$$

where k_C - is the coefficient of linear approximation of the functional transition $C_S \rightarrow C_C$.

Substituting in the relation (8) the values of $u(1)$ and $u(N-1)$ from formula (5), one can get:

$$C_C = k_C k_S k_H k_{Inst} HU\{Exp[-\lambda] - Exp[-\lambda(N-1)]\} \tag{9}$$

This expression shows that an individual's contribution to social capital of a human group depends on the size of the group N, intensity λ, reflecting the degree of egoism and altruism of people in society, the amount of human energy U, reflecting his/her living conditions, intellectual power of people H, reflecting the level of development of society, coefficient k_{Inst} reflecting the effectiveness of the institutional structure of society and linear coefficients k_C, k_S and k_H of functional transitions between the social parameters of society.

Generalizations of the obtained expression will allow to estimate the social capital and asabiya of different scale of human groups, which will be performed in the second part of the work.

Conclusions

1. The activity of any human system is possible only through cooperation with the environment;
2. Cooperation between the environment and the human system is carried out through communication, which can be given a variety of interpretations - information, energetic, etc.;

3. The communication of the human system with the environment is heterogeneous, because it is more intense in its immediate environment and gradually weakens as the purpose of communication moves away from the human system;

4. Communication and cooperation in society create an atmosphere of solidarity and harmony, which, in turn, contributes to the increase of social capital and asabiya, or the ability of people to organize successful joint activities for the stability and development of society;

5. Social capital is manageable, but for effective management of this nature, it is necessary to have quantitative descriptions of various aspects of social life in the form of mathematical models;

6. The core of such a quantitative description may be the heterogeneous nature of communication and contacts of groups of people of different scale with the environment, namely, intensive contacts with the immediate environment of the group and the weakening of these contacts when moving away from it.

Further research

People with certain energy and time resources, as well as limited knowledge, skills and capacities are constantly faced with the problem of ensuring the highest possible quality of life with appropriate levels of safety and prosperity.

From the point of view of the quantitative description of social life, the energy-based approach is the most appropriate, since the energy received by a person in the form of food is relatively easy measurable, which is very important in the sense of the scientific validity and reliability of research methods.

Such an energy-based approach underlies the present work, where quantitative estimates of the corresponding social parameters are carried out at the level of an individual.

In the expanded version, the same energy-based approach is widely applicable to the quantitative description of the behavior of different levels of human systems and large-scale social phenomena.

Further scientific work in this direction implies a transition from mathematical models based on the energy approach at the individual level to a similar quantitative description of social phenomena and processes, taking into account the enormous importance of energy in society.

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Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia). Pavel's publications can be found here: http://www.scribd.com/pbarseghyan and here: http://pavelbarseghyan.wordpress.com/. Pavel can be contacted at terbpl@gmail.com