Mathematical models of human cooperation and the transformation of human capital into social capital

Part 2. Quantitative models of social solidarity (asabiya) and social capital at the level of groups of people and society[1]

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Abstract

One of the main issues in the mathematical theory of human systems is the influence of the size of a group of people on the characteristics of the system, which is otherwise represented as the non-linear scaling problem of human systems.

In the history of mankind there were many cases when stable state unions were created from the unity of tribal communities with a low level of development, and at the same time there are many cases when powerful statehood with a high level of development collapsed in a very short period of time.

In the general theory of systems, these and related questions unite around a common scientific problem, namely, how systems with certain characteristics (in this case, human systems, including: states, societies, organizations, parties, and so on) can be constructed from elements with known properties (in this case, from people) so that they are stable and prosperous.

From this point of view, the simplest case is the formation of groups of people of different scale with given properties, taking into account the nonlinear transformations accompanying such scaling.

The second part of this article is devoted to the study of such integral characteristics of human groups as the energy for their cooperation, social solidarity and social capital.

Introduction

Creating viable social groups of individuals is one of the most important problems of building a social hierarchy in society, a problem on whose solution the stability of society and the quality of life of people depend.

The energy model of interaction at the human level, presented in the first part of the work, makes it possible to evaluate a person's contribution to creating an atmosphere of solidarity or asabiya in society and the formation of social capital [1].

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If we approach the problem of contacts and interaction between people from a quantitative point of view, we will see that a widely used field approach in physics can also be useful for describing people's behavior and activities.

The fact is that each person has contacts and connections with the human environment and creates around himself a certain sphere of influence and corresponding social fields.

These contacts and connections can have different physical, material, financial, informational, psychological and other expressions, which can also be measured in various ways.

The social fields created by man and a group of people are as realistic as the electric, magnetic and thermal fields in physics, and therefore classical field approaches and methods from the realm of physics can be widely used to describe them quantitatively.

The various ways in which people interact and communicate make society a continuous medium of their interaction, an idealized representation of which allows us to consider many aspects of people's social life, including social solidarity and social capital, as part of an quantitative approach.

For the purposes of mathematical modeling, human life and activities are divided into corresponding action flows, which allows each of the above flows to be represented as a separate equation of state, which, in turn, is related to equations of state representing other flows of human actions.

That is, as a result, it turns out that human life and activities as a whole are represented as an algebraic system of equations of state.

In particular, such an approach can be used for the quantitative consideration of such an urgent problem for any statehood as the problem of the relationship between personal interests and public interests of citizens.

If people do not have a rational attitude to this problem, which can be expressed by the absence of even a small but sacrificial act in this direction, this will indicate the weakness of the public thinking of people, and as a result the state itself will also not be effective.

The same applies to the economic development of a country, the effectiveness of its judicial system and other features and indicators of a country at a system level that directly depend on the level of activity of an ordinary citizen.

The basis of all these state-building efforts is human capital, which, with its ability to unite and pursue common goals, creates social capital and an appropriate environment for the successful development of statehood.

The egoistic tendencies of people and their desire for cooperation and conflict play a central role in this process, which quantitative analysis is the main topic of the present work.

A quantitative representation of these phenomena and processes in the form of mathematical models and equations allows us to create a whole set of qualitative and structural patterns aimed at the effective management of society.

The quantitative representation of the social life of a society should be based on mathematical models representing people's tendencies towards selfishness and cooperation, which will clearly reflect the necessary balance between the personal and social interests of people.

It is also important to note here that there is a certain turning point when an ordinary person feels that public interests are his own personal gain, and in the implementation of which statehood with a fair and rational institutional structure can play an important role.

To study group behavior of people, the principle of superposition of mutual influences of people is also important, which, as a rule, is non-linear, but in the first approximation it can be considered linear and used to estimate group parameters of people.

Energy based approach to the quantitative description of people's behavior and activities

The energy based approach to the mathematical modeling of human systems can become one of the bridges connecting social life with its resource base.

It is a well-known fact that the aspirations and goals of people can be achieved only if there is an appropriate energy base, including every minute energy consumption by people, the source of which is the food they take and which underlies their cooperation with other members of society.

In addition, it is a well-known fact that the limited energy and power of people and human systems in general can have a decisive influence on their behavior and activities.

Generally speaking, the limited capacity of people can serve as a starting point for understanding and predicting their behavior and actions.

In the same way, it is possible to explain the impact of the limitations of other resources on social life and related events and their forecasts, including financial, technological, infrastructural and other restrictions.

The energy approach is also very important from the point of view of mathematical modeling of social life in the sense that such an approach provides a direct link between social phenomena and the fundamental principle of energy conservation.

The latter, in turn, will be a useful factor for a deeper and more detailed understanding of social life, using the approaches and methods of classical equilibrium thermodynamics, while clarifying concepts such as the temperature of society and the entropy of people's social life [3].

The fact is that the concept of entropy for the quantitative representation of a balanced life of society is of key and fundamental importance, and the idea that this concept is associated with the degree of disorder of systems and the unpredictability of their behavior requires more reasonable interpretations.

According to the classical definition, if a closed thermodynamic system is in a nonequilibrium state, then subsequent processes lead to an increase in the system's entropy and a gradual transition of the system to an equilibrium state and the maximum entropy value [4].

In this sense, it is logical to interpret entropy not as a measure of system disorder, but as a measure of its equilibrium, which is a completely different and potentially richer interpretation of the concept of entropy.

In addition, according to Boltzmann's definition in statistical mechanics, the greater the number of possible states the system may be in, the greater the entropy [4, 5].

That is, if we approach the question from the point of view of the number of possible states of the system, then the entropy can represent the magnitude of the diversity of the system.

Summarizing what has been said about entropy, it is much more natural and logical to interpret this value not as a measure of disorder in the system, but rather as a measure of the equilibrium and diversity of the system, in particular, of the human system.

If we go back to the quantitative aspects of human behavior and activity, then even if the necessary amount of energy is available, but with limited possibilities for its transmission or distribution, both an individual's physiological depletion and a loss at a higher social level due to limited infrastructure possibilities may occur.

Energy based model of cooperation between people at group level

The energy function $u(x)$ at the individual level, which reflects communication between people and their mutual influence, has great potential to reflect the behavior of groups of people and to evaluate its characteristics of the system level as social capital and other system-wide parameters.

In order to reveal the potential of the function of energy interaction between people $u(x)$, let's consider a simplified model of society in which people are located along the x axis according to the principle of their maximum proximity to each other (Fig. 1).

Such a model represents a group of equally distributed people along the x axis, with a symmetrical and uniform interaction between them, where each person has his own local human environment.

In the model, each individual spends U energy for his needs and for cooperation with other people, half of which is directed to the right side of a person, and the other half is symmetrical to the left side of him.

The figure shows the case when each of the people collaborates with five people on the right and left.

The presented model is based on the simplification of the structure of society, where each person has a local living environment with which he has limited contacts and communication, and, moreover, these contacts and communication are concentrated in close proximity to the person.

This circumstance, that is, that the contacts of people are concentrated in their immediate vicinity, is the cornerstone of the presented model, which, in turn, is connected with the principles of rationality of social life, such as requiring maximum safety of people and a minimum of their energy consumption.

These principles, which are more justified for primitive societies, suggest that, in general, relatively distant connections and contacts for both people and animals are dangerous and energy-intensive.

In parallel with the development of society, which is accompanied by improved conditions for the safety of people and the solution of their transportation problems, their remote contacts with other people are gradually becoming more justified.

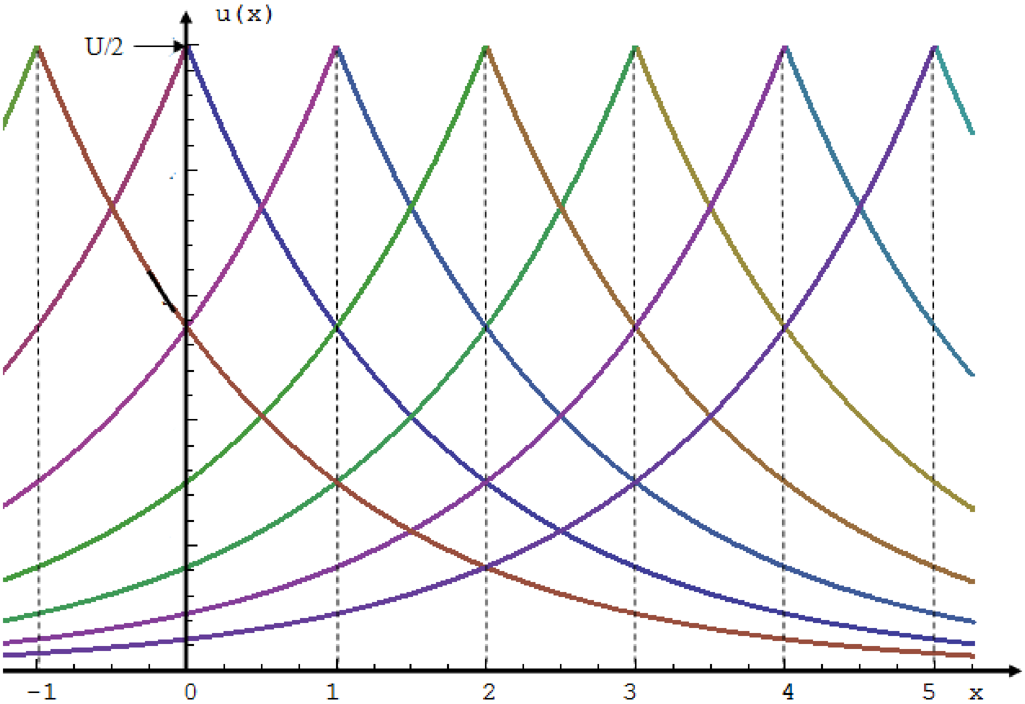

Fig.1 The assignment of an idealized image of relationships between people allows one to quantify the extent of cooperation between people and groups of people at different distances, the potential for cooperation within a group of people, etc.

This will mean that the average function $u(x)$ of one individual in developed societies compared with the same function in primitive societies will fall more slowly (Fig. 2).

Energy based mathematical models of human cooperation

For a quantitative study of group characteristics of people, let's consider a simplified graphical model of their cooperation presented in Fig.3.

In this graphical model, each person has an energy function $u(x)$ of his cooperation with others, which is correspondingly shifted along the x axis.

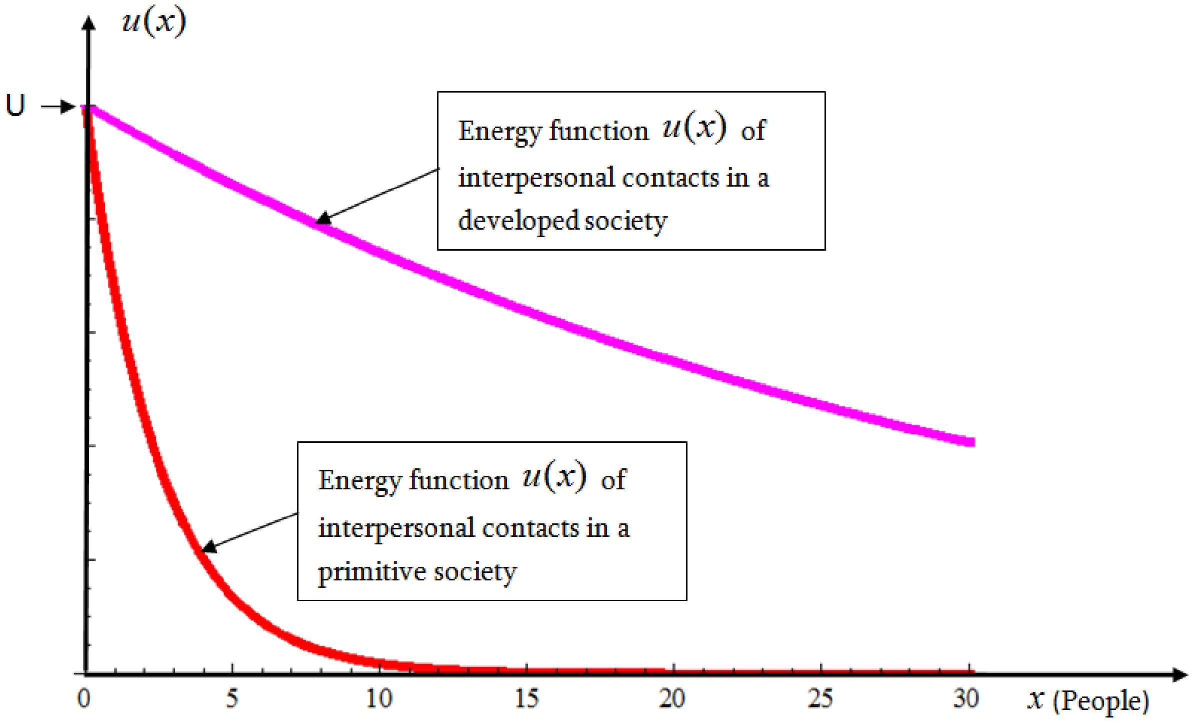

Fig.2 In primitive societies, which are represented in red in the figure, a person communicates with fewer people than in developed societies.

To implement such a shift in the size i of the energy function $u(x)$ along the x axis, it is enough to represent this function as $u(x-i)$.

In particular, in the case of the exponential function $u(x) = UExp[-\lambda x]$ used in the first part of this work, the indicated displacement of this function along the x axis will have the following form - $u(x-i) = UExp[-\lambda(x-i)]$.

In addition, in the graphic model shown in Fig.3, the energy U of an arbitrary i-th person is symmetrically distributed between his right and left neighbors. This means that the analytical expression of the right half of the function $u(x-i)$ of the i-th person will be.

$$u_i(x) = u(x-i) = \frac{U}{2} Exp[-\lambda(x-i)]; \qquad (1)$$

This expression can be used to estimate the amount of energy consumed in a compact group of N people for the personal purposes of the i-th person and for the purposes of his cooperation with others.

Thus, the energy consumption U_{iR} of an arbitrary i-th person for cooperation with neighbors from the group located to the right of him will be

$$U_{iR} = u_i(i+1-i) - u_i(N-1-i) = u(1) - u_i(N-1-i) \qquad (2)$$

Substituting the values of the function $u_i(x) = u(x-i)$ from (1) into the expression (2), we get $U_{iR} = u(1) - u_i(N-1-i) = \frac{U}{2}\{Exp[-\lambda] - Exp[-\lambda(N-1-i)]\}$ (3)

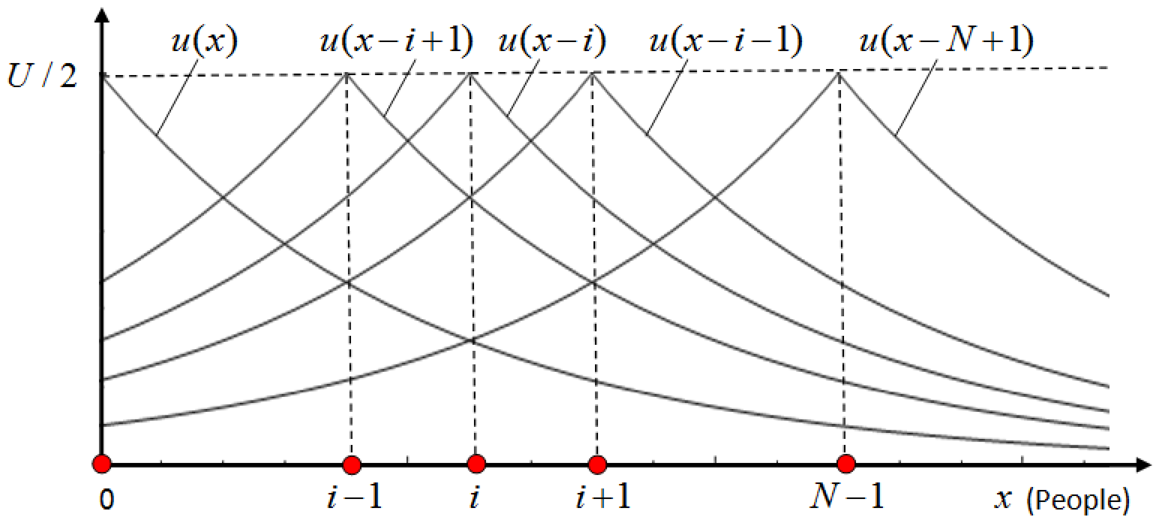

Fig.3 A simplified graphical model of the common life of people, where each i-th person has an energy function $u(x-i)$ for interacting with neighbors, which is shifted along with x axces in accordance with the position of a particular person

Using the principle of superposition, the energy spent on the collaboration of a group of N people can be estimated as follows

$$U_R(N) = \sum_{0}^{N-1} U_{iR} \qquad (4)$$

Taking into account that expression (4) applies only to persons who are located to the right of the giwen point on the x axis and considering that symmetrically the same amount of energy is spent on contacts with people located on the left side of the same point, the total energy expenditure on cooperation inside the group will be

$$U_{IC}(N) = U_R(N) + U_L(N) = 2U_R(N) = 2\sum_{0}^{N-1} U_{iR} \qquad (5)$$

Substituting the value of the quantity U_{iR} from expression (2) into (5), we obtain

$$U_{IC}(N) = 2\sum_{0}^{N-1} U_{iR} = 2\sum_{0}^{N-1}[u(1) - u_i(N-1-i)] \qquad (6)$$

Simplifying expression (6), for the total energy consumption for cooperation, one can obtain:

$$U_{IC}(N) = 2[Nu(1) - \sum_{0}^{N-1} u_i(N-1-i)] \qquad (7)$$

The group of people, in addition to internal relations and contacts, also has external relations with other people, for the realization of which some of its total energy is consumed, for the evaluation of which we will continue to consider the graphical model of cooperation of people shown in Fig.3.

To do this, consider the consumption of energy of a i -th person with neighbors on the right side of the group, which will be equal to the value of the energy function $u(x-i)$ of this person at the point $N-1$ and will be defined as follows

$$U_{iRO} = u(N-1-i) \qquad (8)$$

This means that the energy U_{RO} spent inside the group on contacts and cooperation with the people to their right, according to the law of superposition will be equal to the sum of the energies U_{iRO} spent by all N members of the group, that is,

$$U_{RO} = \sum_{0}^{N-1} U_{iRO} = \sum_{0}^{N-1} u(N-1-i) \qquad (9)$$

Taking into account the fact that the energy consumption U_{LO} of the same human group with people located on the left side on the x axis is symmetrically equal to the energy consumption U_{RO}, the total energy consumption U_O of the group for cooperation with others within the group will be

$$U_O = U_{LO} + U_{RO} = 2U_{RO} = 2\sum_{0}^{N-1} u(N-1-i) \qquad (10)$$

Taking into account that all members of the group spend energy $U_T = N*U$ together, it is possible to estimate the total amount of energy U_I consumed within the group, including the consumption of energy for personal and joint purposes.

$$U_I = U_T - U_O = N*U - 2\sum_{0}^{N-1} u(N-1-i) \qquad (11)$$

Since each person in the group spends $2(\frac{U}{2} - u(1)) = U - 2u(1)$ energy on selfish goals, the whole group's energy expenditure U_{eg} for its own purposes will be:

$$U_{eg} = NU - 2Nu(1)) \qquad (12)$$

From here the energy of collaboration between the members of the group will be

$$U_{IC}(N) = U_I - U_{Eg} \qquad (13)$$

Substituting the values of the energies U_I and U_{Eg} in expression (13), respectively, from expressions (11) and (12), we obtain

$$U_{IC}(N) = 2[Nu(1) - \sum_{0}^{N-1} u_i(N-1-i)] \qquad (14)$$

Thus, taking into account the expression (1), one can obtain the interaction energy $U_{IC}(N)$ between the members of the group

$$U_{IC}(N) = U\{NExp[-\lambda] - \sum_{0}^{N-1} Exp[-\lambda(N-1-i)]\} \qquad (15)$$

Similarly, from expressions (1) and (10) for the energy consumption of the group of people for external contacts, one can get.

$$U_O = U \sum_{0}^{N-1} Exp[-\lambda(N-1-i)] \qquad (16)$$

Together with an increase in the size of a group of people, expressions (15) and (16) can be simplified by moving from sums to integrals, which also allows us to continue the analytical study of the problem.

After such a transition from expression (15) for internal energy consumption for cooperation between people, one can obtain

$$U_{IC}(N) = U\{NExp[-\lambda] - \int_{0}^{N-1} Exp[-\lambda(N-1-x)]dx\} \qquad (17)$$

Integrating this expression, we obtain the energy that the human group spends for cooperation, as a function of the size of the group N and the parameter λ, the graphic image of which is shown in Fig.4

$$U_{IC}(N) = U\{NExp[-\lambda] - \frac{1}{\lambda}(1 - Exp[-\lambda N])\} \qquad (18)$$

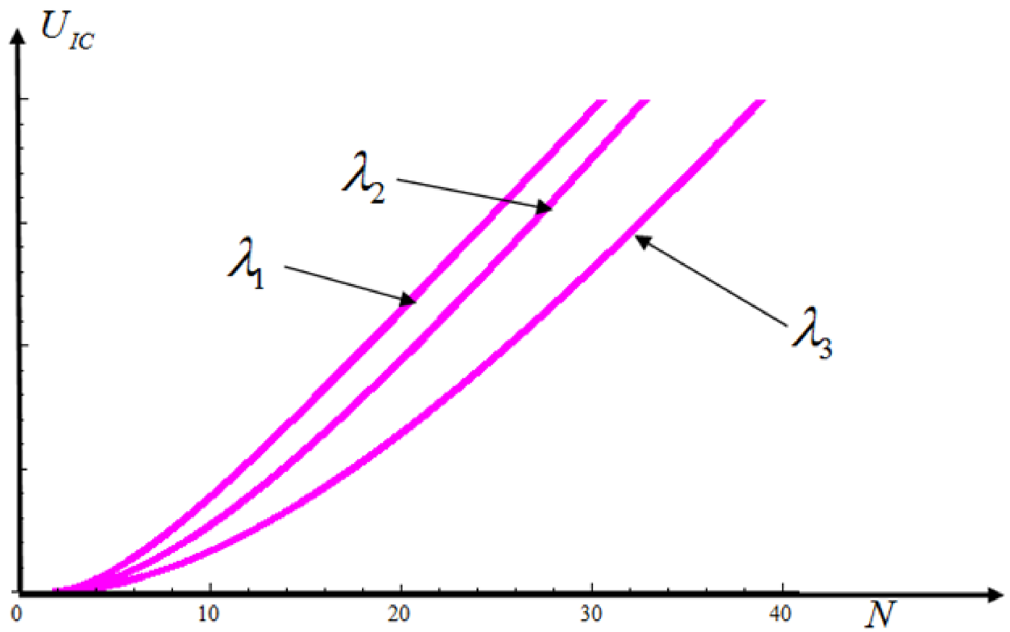

Fig.4 Dependence of energy U_{IC} aimed at the cooperation of people from the group size N and parameter λ ($\lambda_1 > \lambda_2 > \lambda_3$)

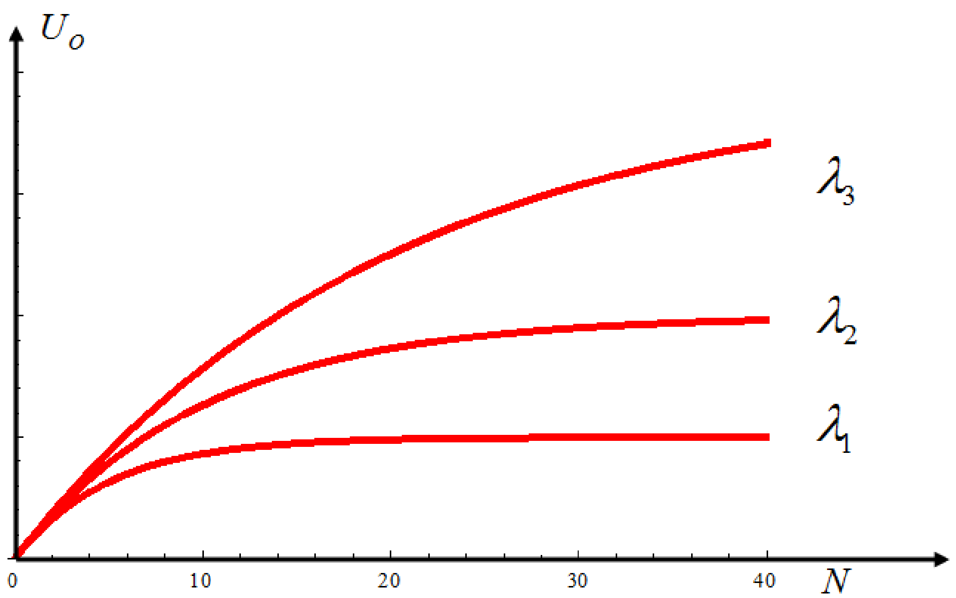

Fig.5 Dependence of energy U_O aimed at working with people outside of the group from the group size N and parameter λ ($\lambda_1 > \lambda_2 > \lambda_3$)

Using the same approach, it is possible from the expression (16) to obtain energy U_O as a function of group size N and parameter λ , spent by the human group on external cooperation in an exponential society

$$U_O = \frac{U}{\lambda}(1 - Exp[-\lambda N]) \ , \qquad (19)$$

graphic image of which is shown in Fig.5.

The internal energy consumption of a group, which is the sum of the energies spent on selfishness and cooperation, in the case of exponential societies can be defined by the expression (11) and (1)

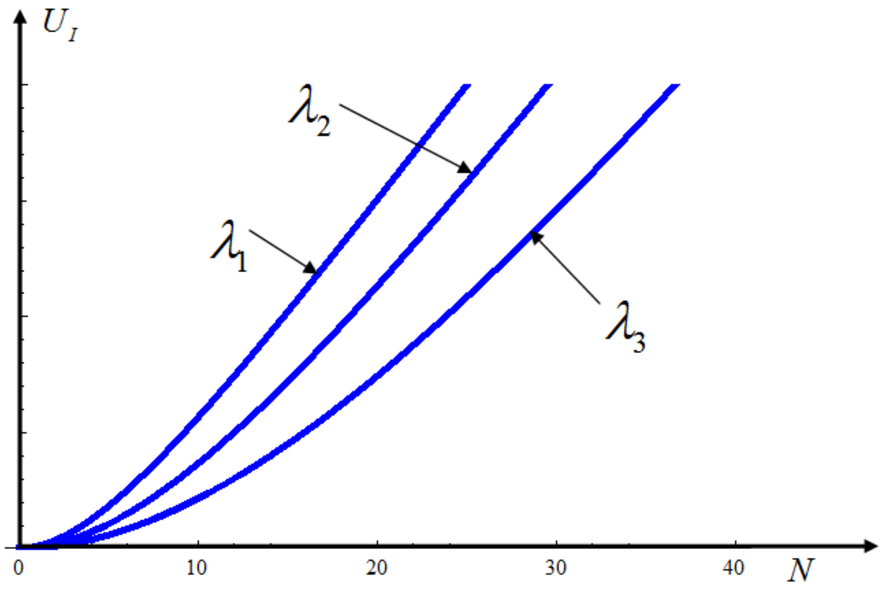

Fig.6 Dependence of energy U_I aimed at the cooperation of people and their personal needs from the group size N and parameter λ ($\lambda_1 > \lambda_2 > \lambda_3$)

$$U_I = U[N - \frac{1}{\lambda}(1 - Exp[-\lambda N])] \qquad (20)$$

The graph of this function is shown in Fig.6.

Mathematical models of people's solidarity, discussed in the first part of work [1], using the results obtained in this paper, can be extended to groups of people of different scale.

Having a variety of energy assessments of the cooperation of people, it is possible with their help to continue the study of the key characteristics of the functioning of human systems, including human solidarity, human capital, social capital, and so on.

In particular, the same results can also be used to study such important problems of public life as the problem of the relationship between egoism and altruism of people, the relationship and balance between one's own interests and the interests of society, which will be presented in the third part of the work.

Conclusions

1. Energy interaction functions, which represent a quantitative model of cooperation between people, create a wide range of opportunities for bottom-up analysis of the characteristics of a group of people of different scale.
2. The method of evaluation of group or integral characteristics of people is based on the principle of superposition of their energy fields.
3. The methods proposed in the work on modeling the behavior and activities of people focus on the key issue, namely the use of the classical field approach, widely used in physics, for a quantitative representation of various fields created by man.
4. In this sense, there must be a discussion about the field of people's influence on others, about moral, power and other fields created by man, and so on.

Future work

Mathematical modeling of cooperation between people and their multi-factor interactions is based on a flow-based approach, in which interactions between people can be represented as some kind of flow, be it energy flow, the spread of people's moods and intentions, etc.

In turn, the indicated flows, be they energy or informational, can be represented by means of differential equations, which will be considered in the third part of the paper.

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Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia). Pavel's publications can be found here: http://www.scribd.com/pbarseghyan and here: http://pavelbarseghyan.wordpress.com/. Pavel can be contacted at terbpl@gmail.com