Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems

Part 1: Mathematical Modeling of New Quality Emergence from Interactions of Qualitatively Different Phenomena by the Method of State Equations [1]

By Pavel Barseghyan, PhD

Abstract

Public life is a stream of different quantitative and qualitative emergencies that are the result of people's actions and activities.

In turn, arbitrary actions and activities of humans can be adequately represented by mathematical models in the form of equations of state.

The equation of state of an arbitrary action or activity combines the various resources needed for people's lives and activities, people's knowledge and skills, and the results of their activities.

The results of people's actions and activities contain a variety of quantitative and qualitativ emergences that have different nature and scale.

In the first part of the article, is considered the mathematical model of the emergance of qualitatively new phenomena as a result of the interaction and integration of various qualitative phenomena.

The following parts of the article will discuss the mathematical models of nonlinear and probabilistic nature of the emergence phenomenon.

Introduction

In recent decades, the quantitative science of human life and activity has been making unprecedented progress for the simple reason that highly developed societies and mankind in general are increasingly in need of it.

---

[1] How to cite this paper: Barseghyan, P. (2020). Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems - Part 1: Mathematical Modeling of New Quality Emergence from Interactions of Qualitatively Different Phenomena by the Method of State Equations; *PM World Journal*, Vol. IX, Issue III, March.

The reason for this is the steep rise of the complexity of human civilization, where the management of public life encounters unprecedented difficulties and challenges of a critical nature, which solutions are no longer effective through traditional means of experience and intuition and often lead to catastrophic and large-scale errors.

The solution to the issue is modern expert systems based on an adequate quantitative description of public processes, which can help critical decision makers in avoiding catastrophic errors with severe consequences.

The core of such expert systems should be the quantitative theory of the activity of humans and human systems.

This theory and the mathematical models underlying it are substantially promoted by the fact that neighboring fields such as theoretical physics, mathematical control theory, mathematical economics, mathematical biology, systems theory, organizational science, etc. have accumulated huge experience in the development of quantitative methods, which can serve as a methodological basis for the creation of serious quantitative science on human behavior and activity.

One of the central problems of quantitative science of human life and activity is the emergence problem, which has been the focus of the leading scientific forces for last decades [1, 2, 3, 4].

The experience of the quantitative sciences mentioned above has shown that the most efficient solutions of the problems related to complex processes and phenomena usually rely on the mathematical description of the fundamental nature of those processes and phenomena.

This means that such a complex phenomenon as emergence, as well as many other human activity related phenomena, can be qualitatively explained and quantitatively described only within the framework of the mathematical theory of human activity of the fundamental nature.

Classification of the emergence phenomenon in terms of mathematical modeling

For mathematical modeling of a fundamental nature, it is also necessary to classify the phenomenon of the emergence of new qualities from different angles.

In this sense, emergence, as a phenomenon, can be **linear** and **non-linear**. In addition, the emergence phenomenon can be considered within the framework of the **deterministic approach**, that is, at the level of average values of the parameters and within the framework of the **probabilistic approach**, that is, at the level of distribution functions of random variables characterizing the emergence phenomenon.

Further, according to another classification, the emergence can be **weak** and **strong**, a circumstance that is presented in more detail in the scientific literature [2, 4].

If we look at the emergence classification from the perspective that they are the result of human activity, then similar to human activity emergence also can be **homogeneous** or **heterogeneous** [6].

Emergence, like any other phenomenon related to people's lives and activities, can be independent of the time factor, that is to say, **time invariant** and **time dependent**.

Accordingly, we can deal with the phenomena of **static emergencies** and **dynamic emergencies**, whose mathematical description in static cases will have an algebraic character, and in dynamic cases it will be based on differential equations.

What is the equation of state of human activity?

In general, given that any emergence in society is the result of human activity, this phenomenon can be studied within the framework of the mathematical theory of human systems [5, 6].

For that purpose, let's consider the basic principles and results of the above theory for the construction of mathematical models of emergence phenomenon.

At the core of the quantitative description of human life and activity is the mathematical model of the sequence of homogeneous actions of humans in the form of state equations [7, 8, 9, 10].

The meaning of these equations is that the fundamental requirement for the continuity of the flow of life at every step forces people to perform actions in accordance with the current requirement of life, during the execution of which people seek to maximize the probability of their successful completion.

Successful human performance of an action implies a certain balance between the difficulty of the action and the abilities of the human being.

The mathematical representation of this balance is the equation of the state of homogeneous activity of humans.

If a person during the time period T has successfully performed n actions, each of which has a magnitude w and a degree of difficulty D, then the state equation of human activity will be [5, 7]

$$T * P = n * w * D, \tag{1}$$

where P is the efficiency or productivity of a person to perform a given type of activity, or the number of successfully completed actions in unit time.

At every step of life, the equation of state of a person's activity reflects the relationship between the person's abilities and the difficulty of the task he is currently performing and, in particular, the balance between them.

The above also assumes that a person has performed some number of actions I in unit of time, out of which only P actions has been successfully completed.

The magnitude I is the total intensity of human activity, and P - is the intensity of successful activity, and these parameters are related with each other by the following functional relation [11].

$$P = \frac{I}{1+at_a} \ , \tag{2}$$

where a is the intensity of human-made errors and t_a is the average time to correct or eliminate one error.

In addition, the productivity P and the intensity of human activity I due to the low motivation of people M $(0 < M < 1)$ can be significantly lower than their maximum possible values P_{Max} and I_{Max}, i.e.

$$P = M * P_{Max} \tag{3}$$

and

$$I = M * I_{Max} \ . \tag{4}$$

Combining the expressions (2) and (4) one can get the following equation of the state of homogeneous activity of humans

$$T * M * I_{Max} = (1 + at_a) * W * D, \tag{5}$$

where $W = n * w$ - is the sum value of human activities in the period of time T.

If we are talking about the joint activity of not one person, but N people, then the equation of state will be

$$N * T * M * I_{sMax} = (1 + a_s t_{as}) * W * D, \qquad (6)$$

where the values I_{sMax}, a_s and t_{as} will relate already to the human system.

If the human activity is such that the W_0 portion of the size W is known from the previous activity and can be reused, then equation (6) will look like this.

$$N * T * M * I_{sMax} = (1 + a_s t_{as}) * (W - W_{0)} * D. \qquad (7)$$

This equation can have many practical applications, as the reuse of the results of their previous work by human systems is an integral part of people's lives and, in particular, an integral part of any project or activity planning.

The speed of people's activities

From the equations of state of people's activity can also be obtained equations of state for the speed of their activity.

In particular, if we divide the two parts of equation (6) by the time period T and, given that the $\frac{W}{T}$ ratio is the average speed v_{av} of human activity in a given period of time, we can obtain a new equation of state.

$$N * M * I_{sMax} = (1 + a_s t_{as}) * v_{av} * D\ . \qquad (8)$$

From here one can get the average speed of human activity

$$v_{av} = \frac{N*M*I_{sMax}}{(1+a_s t_{as})*D}\ . \qquad (9)$$

The relationship between equations of state with a quantitative description of emergence as a phenomenon

Speaking about the emergence in a broad sense, one can interpret them as the results of actions and activities of people of any complexity and difficulty, whether it is the result of their simple biological functions or the result of complex social activities.

This means that by saying emergence we can understand both the emergence of new qualities through the interaction and integration of old qualities, and the repetition of the emergence of old, already known qualities.

Even if talking about an emergence, we understand only the appearance of a new qualitiy, then from the point of view of mathematical models based on state equations this is not important, since the models of the appearance of a new quality and the repetition of an old quality in the structural mathematical sense are the same.

That is, the method of equations of state in a universal way can describe the results of any human activity, whether it is the birth of a new quality or a repetition of an old one.

The peculiarity of the equation of state from the point of view of the emergence is that some parameters in this equation reflect the properties of people, and the other part - the properties of the tasks implementing by them, and the combination of these two qualities through the equation of state describes a new quality - human activity.

That is to say, the combination of two different qualities, that is, life-driven demand and the supply of human abilities provided with various resources and tools gives birth to a new quality - human activity.

This means that each specific manifestation of the emergence phenomenon can be quantitatively represented by the equation of state of human activity.

The quantitative description of a qualitative phenomenon like emergence is based on the fact that the W size of human activity can be directly related to the R results of that activity, in particular with the generation of new qualities [7, 8].

Relying on the logic that the larger the size W of human activity, the more results R it can produce, the functional relation $R(W)$ can be approximated with a simple linear law.

$$R = k_R W, \qquad (10)$$

or

$$W = \frac{1}{k_R} R, \qquad (11)$$

where k_R is the coefficient of propotionality.

Substituting the value of W from (11) into equation (6) and solving it for the result of people's activity R, one can get

$$R = \frac{k_R * N * T * M * I_{sMax}}{(1 + a_s t_{as}) * D} \qquad (12)$$

Expression (12) is the simplest mathematical model of the results of human activity and emergence, which is the basis for the development of more advanced quantitative models that reflect the lives of people and society.

Mathematical model of the emergence of a new quality from the interaction and integration of different qualities with equations of state

The main problem in the mathematical modeling of the emergence phenomenon is the quantitative description of the process of generating a new quality by combining or integrating different qualities.

Such a quantitative description, in principle, is possible if there are certain preconditions associated with quantitative representations of the life and activity of people using equations of state for that purpose [9].

For this purpose, it is first necessary to represent the old qualities by means of separate equations of state, each of which represents the emergence or occurance of the old qualities as a result of human activity.

In order to obtain a new quality from the merging or integration of the old qualities, in addition to the quantitative representation of the old qualities with equations of state, it is also necessary to quantify the interaction of the old qualities with the possible functional links between the parameters of their equations of state.

An example of this kind of emergence of a new quality is discussed in [9], where the breakdown of the sequences of actions characterizing the life path of people into the functionally different subsequences of actions allows quantitatively, that is, using equations of state, to describe different aspects of the life and activities of people that are also sources of various qualities.

Thus, as is done in the above mentioned work, people's lives can be divided into two subgroups of actions, that is, the sequence of biological actions and the sequence of social actions, each of which in arbitrary time period will have its own equation of state.

To make the problem more specific, one can separate the human labor activity from social activity, and the actions associated with human nutrition from the sequence of biological actions, on the basis of which one can build a model of one of the closed cycles of human life, the meaning of which is as follows.

The sequence of actions associated with human labor has two consequences: the first one is that the person loses the energy received as a result of nutrition, and, in addition, receives a reward for the work that he/she performs, and then part of this reward goes to purchase food.

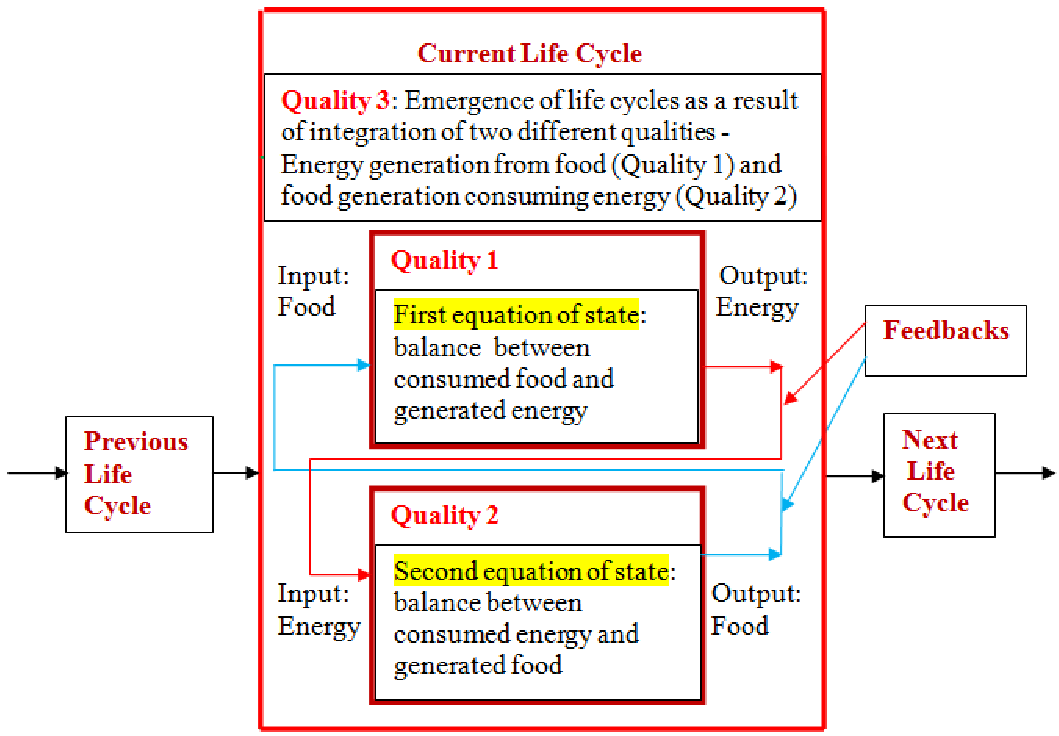


Fig. 1 Continuous chain of human life cycles

Then, by eating the food, a person regains his energy and thus closes the local life cycle.

In this example, the two phenomena that are qualitatively different from one another, namely, working and losing energy and recovering it from eating, complement each other and create a new quality in the form of normal course of human life.

From a quantitative point of view, this is due to the fact that each of the qualitatively different phenomenon, human labor associated with the loss of energy and eating food to restore energy, is presented as a separate equation of state, and the links between the obtained equations is carried out through feedbacks. This makes the constructed mathematical model in the form of a system of equations of state a means for adequately representing a new quality - the normal life process of people (Fig.1).

Emergence phenomenon and the method of state equations

The emergence of new qualitative phenomena as a result of human activity is an integral part of the life of human society, which daily has many and diverse manifestations.

The life of the human system, be it a person, family, organization, state, or civilization, can ultimately be broken down into sequences of different size and content of actions and their combinations, each of which can represent or produce a certain quality.

Mathematically, each of these sequences of human actions can be represented by a state equation, which combines the resources, knowledge and skills needed to perform a particular type of activity and produces results in the form of a certain qualitative phenomenon.

In their turn, through the interaction and integration of different qualitative phenomena or their equivalent actions flows, completely new qualitative phenomena can emerge or be generated.

The peculiarity of the method of state equations is that makes it possible to represent different qualities in the form of different equations of state.

As for the semantic integration of different qualities, it is done through the feedbacks between different equations of state and the possible functional links between their parameters.

If each of the component qualities of an integral qualitative phenomenon is represented as a separate equation of state, then the new quality is mathematically represented as a system of state equations, where the number of equations is equal to the number of component qualities.

Some typical examples of the quantitative representation of the emergence phenomenon using state equations

To illustrate the widespread applicability of the state equation method, let us consider some typical examples of generating new qualities based on the description of structure and activities of human systems.

1. State power as a result of interaction and integration of different branches of power

The main modern model for the governance of a state is the separation of powers into different branches, which have different responsibilities and which do not have the right to interfere with the functions of other branches of power [12].

The purpose of the separation of powers is to prevent the concentration of power on one of the branches of power through mutual restraint and counterbalance.

The quantitative representation of the separation of powers by the method of state equations has the form of a system of equations of state where each equation describes the activity of a branch of power.

If we look at the typical case of the separation of powers, when it is divided into legislative, executive and judicial branches, then the mathematical model of such a system of powers in arbitrary period of time T will have the following form

$$N_L * T * P_{sL} = W_L * D_L \,, \qquad (13\text{-}1)$$

$$N_E * T * P_{sE} = W_E * D_E \,, \qquad (13\text{-}2)$$

$$N_J * T * P_{sJ} = W_J * D_J \,. \qquad (13\text{-}3)$$

In these equations, the L, E, and J indices mean Legislative, Executive, and Judicial, respectively.

From a semantic point of view, equation (13-1) means that the legislative branch, having staff N_L during the period T with efficiency P_{sL} overcoming the difficulties of D_L, carries out the activity W_L, which generates laws.

Equation (13-2) likewise represents the activity of the executive branch of power, which, within the laws, makes or produces decisions and implements governance.

As for equation (13-3), it represents the country's judicial system, which again, within the framework of the law, generates decisions regarding violations of the law.

All the equations in the system of equations (13) are interconnected with each other, since many of their parameters have functional relationships with each other, as a result of which they form a functional whole.

These equations represent three different qualities of power, from the interaction and integration of which a new quality is born - state power.

A more detailed mathematical model of the separation of powers, which also takes into account various motivations of the branches of power related to political or other considerations, and errors made in the course of their activities, will look as follows.

$$N_L * T * M_L * P_{sL} = (1 + a_{sL} t_{asL}) * W_L * D_L \ , \qquad (14\text{-}1)$$

$$N_E * T * M_E * P_{sE} = (1 + a_{sE} t_{asE}) * W_E * D_E \ , \qquad (14\text{-}2)$$

$$N_J * T * M_J * P_{sJ} = \left(1 + a_{sJ} t_{asJ}\right) * W_J * D_J \ . \qquad (14\text{-}3)$$

In their totality, the systems of equations (13) and (14) in various degrees of detail represent the mathematical model of a democratic power.

The method of state equations of human systems also gives an oportunity to describe quantitatively the emergence of dictatorial, authoritarian, military, colonial, and other forms of power.

For a complete quantitative representation of the country's activity by the method of state equations, the equations of state of the people and the opposition to the above three equations should be added, which will also have functional connections with the basic three equations.

2. The emergence of people's complaints, protests, and change of power in society

If we add to the system of equations (14) the state equation of the people and the state equation of the opposition and their functional relationships with the state equations of the branches of power, one can get a new system of five equations of state through which it is possible to quantify many phenomena of public life, including discontent and complaints of people and, as a result, the change of power.

$$N_L * T * M_L * P_{sL} = (1 + a_{sL} t_{asL}) * W_L * D_L \ , \qquad (15\text{-}1)$$

$$N_E * T * M_E * P_{sE} = (1 + a_{sE} t_{asE}) * W_E * D_E \ , \qquad (15\text{-}2)$$

$$N_J * T * M_J * P_{sJ} = \left(1 + a_{sJ} t_{asJ}\right) * W_J * D_J \ , \qquad (15\text{-}3)$$

$$N_O * T * M_O * P_{sO} = (1 + a_{sO} t_{asO}) * W_O * D_O \ , \qquad (15\text{-}4)$$

$$N_P * T * M_P * P_{sP} = (1 + a_{sP} t_{asP}) * W_P * D_P \ . \qquad (15\text{-}5)$$

Here, in equations (15-4) and (15-5), the O and P indices mean opposition and people.

Equation (15-4) represents the activities of the opposition, as a result of which pressure is generated on power, and (15-5) represents the activities of the people.

Depending on the quality of the government's activities, the scale and consequences of its errors, the people may be dissatisfied and complained of, which in turn may be exacerbated by the activities of the opposition.

This means that the system of equations of state (15), which is generally the mathematical model of a state activity, can describe different levels of people's dissatisfaction and, as a result, the change of power.

3. The emergence of the coalitions of human systems

The unification of human beings' abilities and efforts contributes to their aspirations to ensure the safety and quality of life.

In particular, the conflict of human systems forces them to unite into different coalitions and unions.

One of the most important security guarantees is the balance of opposing forces and coalitions are a means by which such balance can be achieved.

If for the sake of simplicity we take the case of the three conflicting human systems, then each of them has a state equation, which produces pressures and dangers for the other two [10].

In the process of conflict, the first human system creates O_{12} and O_{13} threats to the second and third systems.

If the risks, related to the threats O_{12} and O_{13} are significantly greater than the O_{21} and O_{31} response risks, then the only rational safeguard for the second and third systems is to temporarily forget their confrontation and unite their forces to achieve a balance of power with the first human system.

The described phenomenon can be easily represented by equations of state, which would be the mathematical model of the coalition emergence due to the confrontation of human systems.

Conclusions

1. Individual actions of humans as well as their activities of any scale can be quantitatively represented by equations of state.

2. The equation of state of a human system is not a formula by which calculations and estimations can be made, since it contains in a non-explicit way a number of nonlinear functional relationships that can be extracted from equation only by means of additional terms, just as it is done in physics.
3. The equation of state of the human system of arbitrary scale can be interpreted as a phenomenon of quality formation, whether it is a creation of a new quality or the repetition of an already known quality.
4. Similarly, the birth of any emergence or new qualitative phenomenon related to public life and activity of humans can also be quantitatively represented by equations of state.
5. Qualitatively new phenomena may arise as a result of individual actions of humans and as a result of the interaction and integration of their activities.
6. An arbitrary equation of state in itself can be considered as the result of emergence, where the balance that generates the equation of state is built on the basis of interaction and integration of two qualities, namely: the next demand of life process and the reaction of people to this demand.
7. It is also important here that both the demand of life and the actions of people in the form of supply themselves are also the results of human activities with their own equations of state.
8. A social phenomenon of an arbitrary scale, be it an organization, a state or a political party, can be interpreted as an emergence that is the result of interaction and integration of qualities of different nature.
9. In this sense, the mathematical model of the new higher-level emergence will be a system of equations of component qualities, the integrity of which is ensured by positive or negative feedbacks between different state equations.
10. The very structure of the mathematical model of the activity of the human system in the form of systems of state equations with feedbacks makes that approach suitable for solving the problem of emargence of any complexity and scale.

Continuation of work

The mathematical models discussed in this paper can qualitatively and quantitatively adequately explain the phenomena of linear emergence, but they cannot be used to quantify the emergence of non-linear and probabilistic nature.

On the other hand, the linear emergence methods presented in this work, based on the basic balances of people's lives and activities, can serve as a basis for the improvement of mathematical models in different problem areas.

From such problematic areas, the following will be presented in parts two and three of this article:

- Development of mathematical models of nonlinear emergence by the method of state equations,
- Development of mathematical models of probabilistic emergence and risk assessment of non-emergence based on the method of state equations,
- Development of dynamic or time-dependent mathematical models of emergence based on the method of state equations.

References

1. Goldstein J. Emergence as a Construct: History and Issues. February 1999 Emergence 1(1):49-72. https://www.researchgate.net/publication/243786253_Emergence_as_a_Construct_History_and_Issues

2. Bedau, Mark A. (1997), Weak Emergence. http://people.reed.edu/~mab/papers/weak.emergence.pdf

3. Hodgson G. The Concept of Emergence in Social Science: Its History and Importance. Emergence: Complexity and Organization. 2000. https://journal.emergentpublications.com/article/the-concept-of-emergence-in-social-science/

4. Generative Emergence: A New Discipline of Organizational, Entrepreneurial, and Social Innovation 1st Edition, by Benyamin Lichtenstein, 2014.

5. Pavel Barseghyan (2005) **Mathematical Theory of Project Management**. Information Technologies and Management (pp. 21 - 81), (in Armenian language).

6. Pavel Barseghyan (2009) "**Problems of the Mathematical Theory of Human Work (Principles of mathematical modeling in project management)".** *PM World Today* – August 2009 (Vol XI, Issue VIII). 24 pages. http://www.scribd.com/doc/113988182/Problems-of-the-Mathematical-Theory-of-Human-Work-Principles-of-Mathematical-Modeling-in-Project-Management

7. **Pavel Barseghyan (2017) Mathematical Modeling of People's Behavior (Elements of the Mathematical Theory of Human Systems) - 39 pages. (in Russian language).** http://www.iarex.ru/sociology/53498.html

8. Pavel Barseghyan (2017) **“Elements of the Mathematical Theory of Human Systems Part 1: Assessment of the Results of Human Actions and Activities Based on the Method of State Equations ”** *PM World Journal Volume 6, Issue 12 December 2017* – 13 pages. https://pmworldlibrary.net/wp-content/uploads/2017/12/pmwj65-Dec2017-Barsegyan-mathematical-equation-of-human-systems-part1-featured-paper.pdf

9. Pavel Barseghyan (2018) **"Elements of the Mathematical Theory of Human Systems Part 2: Structural Mathematical Models of the Life of Humans Based on the Method of State Equations"** *PM World Journal Volume 7, Issue 1 January 2018* – 12 pages. https://pmworldlibrary.net/wp-content/uploads/2018/01/pmwj66-Jan2018-Barsegyan-mathematical-models-of-life-of-humans-featured-paper.pdf

10. Pavel Barseghyan (2018) **"Elements of the Mathematical Theory of Human Systems Part 5: Quantitative analysis of the benefits and losses of two human systems in the regime of confrontational equilibrium of their interrelations "** *PM World Journal Volume 7, Issue 4 April 2018* – 13 pages. https://pmworldlibrary.net/wp-content/uploads/2018/04/pmwj69-Apr2018-Barseghyan-elements-of-mathematical-theory-of-human-systems-part5.pdf.

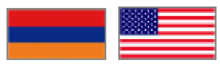
11. Pavel Barseghyan (2010). “**Dynamic Mathematical Models of Human Work (Differential Equations of Human Labor)”.** *PM World Today* – October 2010 (Vol XII, Issue X). 15 pages. http://www.scribd.com/doc/114757539/Dynamic-Mathematical-Models-of-Human-Work-Differential-Equations-of-Human-Labor

12. **Separation of powers**. Wikipedia. https://en.wikipedia.org/wiki/Separation_of_powers.

About the Author

Pavel Barseghyan, PhD

Yerevan, Armenia
Plano, Texas, USA



Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia).

Pavel's publications can be found here: http://www.scribd.com/pbarseghyan and here: http://pavelbarseghyan.wordpress.com/. Pavel can be contacted at terbpl@gmail.com