PM World Journal (ISSN: 2330-4480)
Vol. IX, Issue IV – April 2020
www.pmworldjournal.com

Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems

Part 2: Nonlinear Mathematical Models of New Quality Emergence [1]

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Abstract

The second part of the article discusses the nonlinear functional relationships between system-level parameters of human systems underlying the deterministic emergence phenomenon.

The discussion is based on equations of state, which may be linear or nonlinear depending on the conditions of human activity.

Nonlinear relationships in equations of state are inherently divided into two categories.

The non-linearities of the first category are those contained in linear equations of state.

The second category of non-linearities is due to the limited capabilities of people and human systems, which are described by non-linear equations of state.

Introduction: The progress of human society occurs mainly in the nonlinear field of human activity

The requirements for mathematical models of any phenomenon or process are by their nature divided into two types: qualitative or behavioral requirements and quantitative requirements.

The first step towards mathematical modeling in terms of the quantitative description of the fundamental nature of phenomena and processes is to ensure qualitative or behavioral adequacy of the model.

By the behavioral adequacy of a mathematical model is meant a non contradictary reflection of the functional relationships between the characteristics of the object under study.

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[1] How to cite this paper: Barseghyan, P. (2020). Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems - Part 2: Nonlinear Mathematical Models of New Quality Emergence; *PM World Journal*, Vol. IX, Issue IV, April.

If the mathematical model is in a behavioral correspondence with the modeled phenomenon or process, then only after that one can proceed to ensure quantitative characteristics of the model.

The mathematical models considered in the first part of the article, which dealt with human activity and emergence phenomena, were qualitatively adequate only for linear cases, that is, when the duration of the actions or activities of people was directly proportional to their degree of complexity or difficulty [1, 2].

As a first approximation, this linear dependence can be obtained directly from the equation of state of an individual's activity

$$T = \frac{W}{M * P_{Max}} D. \tag{1}$$

This means that in the first part of the article, when developing a mathematical model of human activity and the emergence phenomenon, without mentioning it specifically, it is assumed that the above linear relationship is true for all levels of complexity or difficulty of human actions and activity.

The same is assumed in the case of a more detailed quantitative representation of the emergence phenomenon, which also takes into account human errors, in which the functional dependence T (D) has the following form [2].

$$T = \frac{(1 + a_s t_{as}) * R}{k_R * N * M * I_{sMax}} D. \tag{2}$$

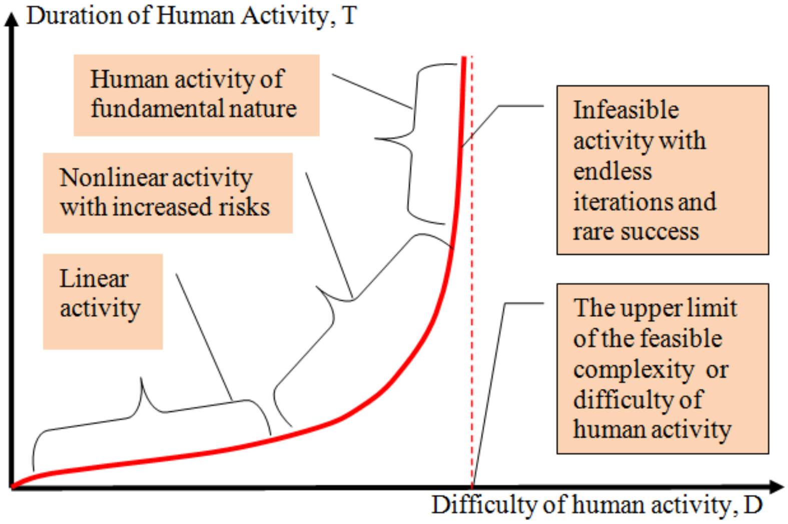

Fig.1. Functional relationship between human activity's difficulty and duration (If the difficulty of human activity increases and approaches to its upper limit, then its duration increases sharply because of endless iterations and gradually the activity of humans becomes infeasible)

As can be seen from expression (2), the dependence T (D) formally has a linear form, but a deeper examination of the issue shows that this expression contains a number of nonlinear dependencies that can only be revealed using additional extreme conditions that reflect the goals of people's activities [3.4].

An analysis of the nonlinear relationships contained in the linear equation of state will be discussed below, and now we turn to the consideration of another important source of nonlinearity of the emergence phenomenon.

The upper limit of the difficulty of human actions as a source of nonlinearity of the emergence phenomenon

It is a known fact that people's physical and mental abilities are limited from above, and the above linear dependence accurately reflects people's behavior to some degree or level of complexity or difficulty in their actions and activities (Fig. 1).

As can be seen from the figure, the T (D) curve has a pronounced nonlinear character, which can be used as a real basis for constructing nonlinear mathematical models of emergence, using for that linear mathematical models of this phenomenon.

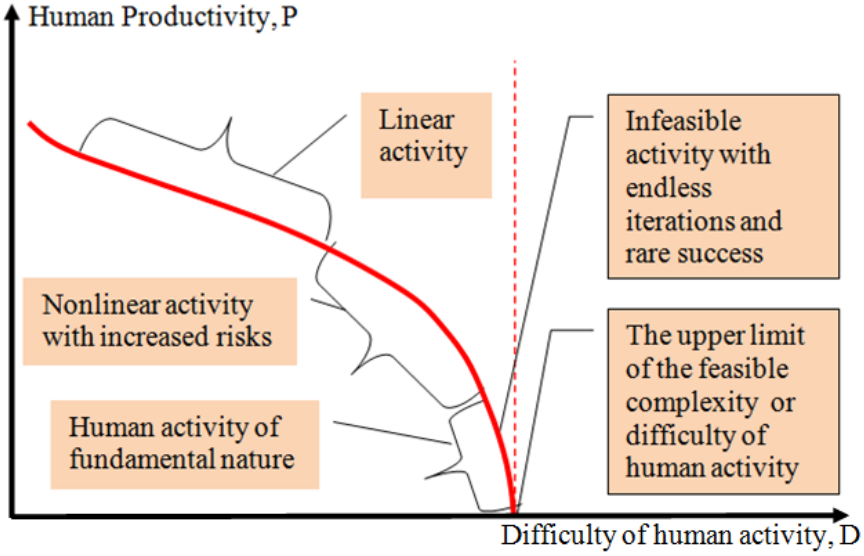

Fig. 2 Non-linear functional relationship of the effectiveness of human activity with the average difficulty of the actions

Another feature of the expression of the duration of human activity (1) is that the effectiveness of P_{Max} or simply the productivity of a person depends on the difficulty of his actions D, the graphic form of which is shown in Fig. 2.

It should be noted that a quantitative representation of the upper limit of people's ability to overcome difficulties makes it possible to take into account such important factors as human learning and training in mathematical models of human activity.

The impact of learning and training on the characteristics of people can be seen in Fig. 3, which shows the dependence of the duration of their activities on the level of difficulty of their actions before and after learning and training.

The upper limit of resistance to external pressure as a source of non-linear losses of the human systems

An example of the nonlinearity of the behavior of people and human systems in a conflict situation is the dependence of their losses on the pressure exerted on them by an opponent [7].

This functional dependence is a generalization of the dependence shown in Fig. 1, since time loss is a special case of generalized loss, and the difficulty of any activity can be considered as an analogue of the pressure exerted on the human system (Fig. 4).

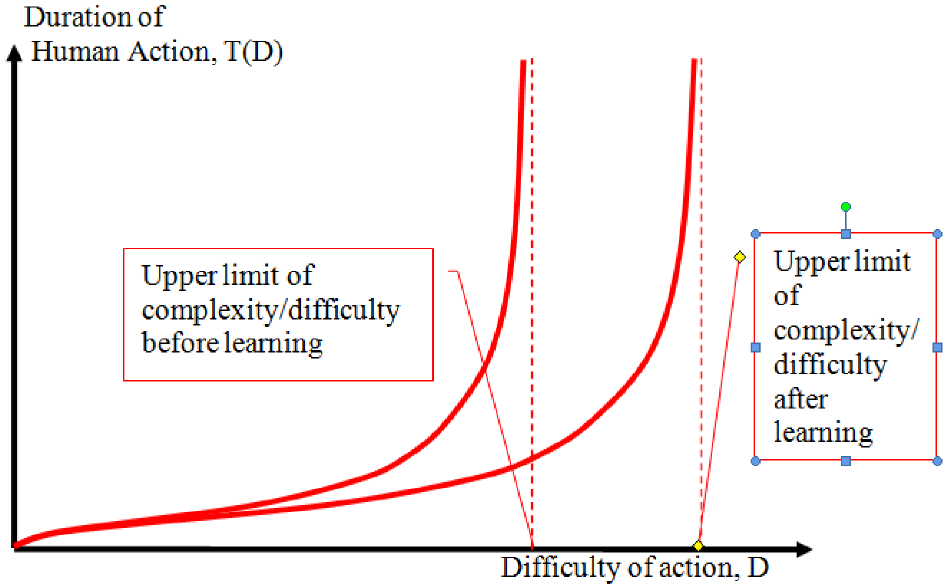

Fig.3 Dependence of the duration of human activity on the difficulty of his actions before and after learning or training

Pressures of varying nature and magnitude on human systems can be sources of a variety of loss-making emergencies, including financial losses, defeat in the war, loss of stability, and so on.

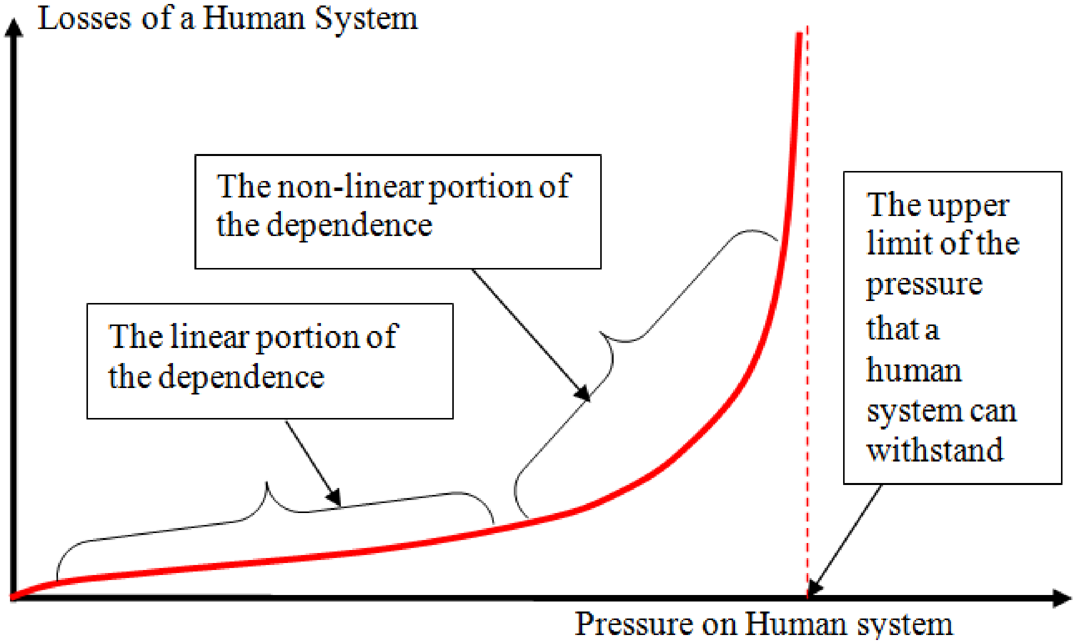

Fig. 4 An arbitrary human system can be characterized by its ability to withstand external pressure and the upper limit of that capacity

In the aforementioned work [7], linear and nonlinear approximations of this curve are used to construct mathematical models of conflicts of human systems.

The upper limit of human hunger as a source of non-linear behavior of human systems

The level of hunger of people also has an upper bound, after which a person passes into the zone of starvation.

People's low quality of life and associated food shortages and an increase in hunger initially lead to inconveniences, and its subsequent increase in protests, complaints, and even more radical actions.

For the sake of a more effective quantitative description of this phenomenon, it is convenient to observe the degree of satiety instead of the degree of hunger, in which case the graphical picture of that phenomenon will look like in Fig. 5.

If instead of starvation, the horizontal axis is given the meaning of satiety, as is done in Fig. 5, then it is already necessary to introduce the concept of the lower limit of human capabilities, which in this case means the lower limit of satiety, that is, hunger.

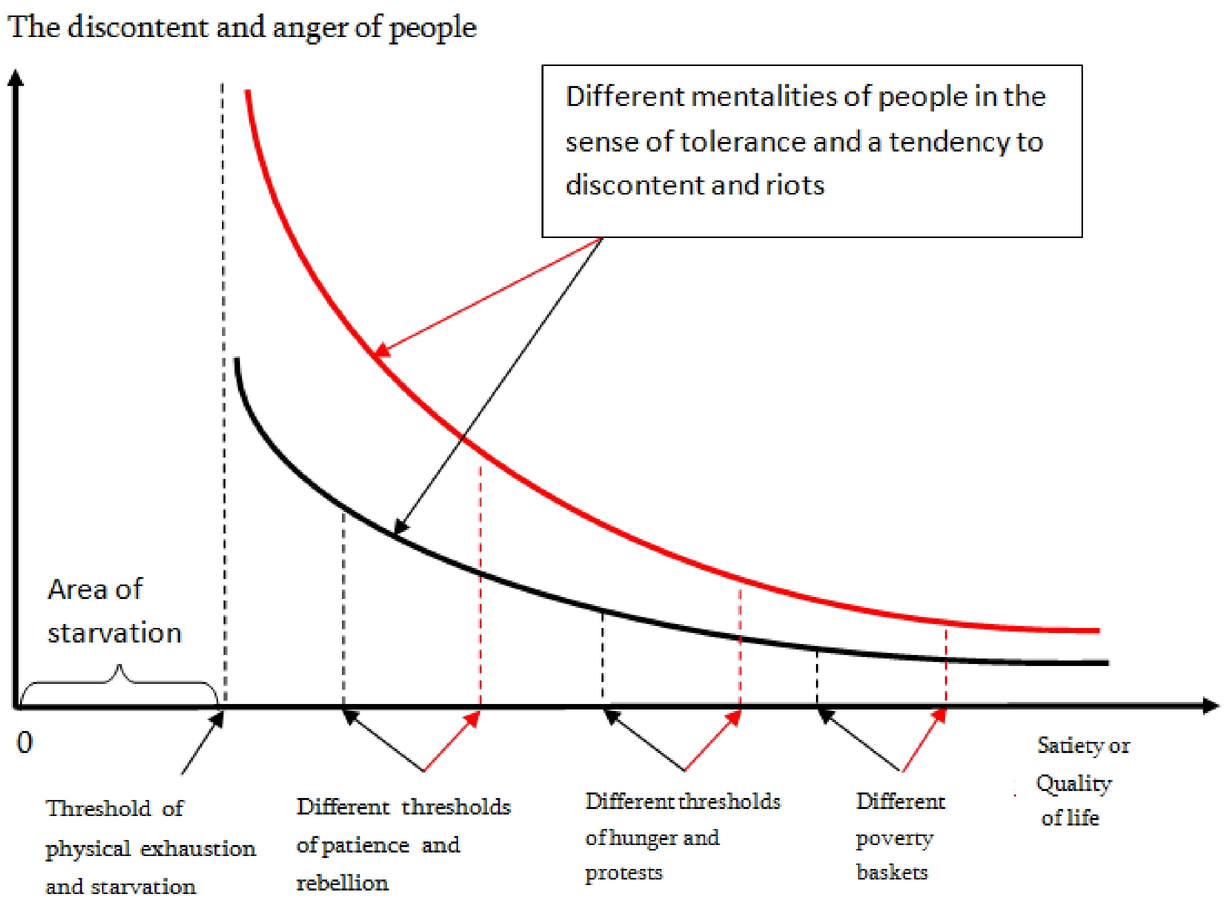

Fig.5 Non-linear dependence of people's dissatisfaction with their degree of hunger

A mathematical description of this phenomenon is again made by the method of equations of state, a detailed description of which can be found in [1].

Two types or categories of nonlinear emergence

Thus, the presented examples show that the phenomenon of nonlinear emergence associated with human activity can be divided into two types or categories.

The first category of emergence is associated with non-linear functional relationships between the parameters of linear equations of state, and the second category of non-linear emergence is associated with the characteristics of the activity of human systems in areas close to the upper limit of their capabilities.

All forms of nonlinear emergence can be quantitatively described by the method of state equations, to which the rest of the article is devoted.

Extraction of nonlinear laws of emergence of the first category from a linear equation of state by its joint solution with the condition of an extremum

Each equation of state of human activity contains an infinite number of possible trajectories that characterize the life process.

People, depending on what purpose they pursue in their actions, choose one of the infinite number of possible trajectories of life indicated above.

That is, if we consider the problem from the point of view of emergence as a phenomenon, then the equation of state is one quality that serves as a generator of possible trajectories of life, and the goal of a person in the form of an objective function is another quality that, as a filter, selects from an infinite number of possible behavioral trajectories of life only those that are consistent with the purpose of the person.

This means that as a result of the interaction and integration of two qualities - the equation of state of human activity as a generator of life trajectories and the objective function of a person as a filter, a life trajectory is born in the form of a chain of decisions and actions that pursue a specific goal.

To illustrate this, let us consider a usual problem of people's business planning that has a given complexity C [3,4,5,6].

In this case, the equation of activities of a certain group of people would in the simplest case look like this:

$$T * N * P = C. \tag{3}$$

The meaning of this equation of state is that a group consisting of jointly acting N persons with a productivity P during a period T can perform work with complexity C.

The purpose of planning such an activity is to select the values of N and T in terms of the cost and risk of doing business.

In solving this problem, one must first take into account the fact that the values P, N and T are related to each other by nonlinear functional relationships [1, 3].

For example, it is well known that the effectiveness of the human system depends on the number of people working together, since with the increase in the number of people, the time required for their communication, coordination and control of their activities increases, which at the level of human groups first leads to an increase in productivity P, and then to its decrease.

There is also an obvious non-linear functional relationship between the number of people N and the duration of their activity T, but this relation is not hyperbolic, as it may seem from expression (3), since with the change of N, productivity P also changes.

Regarding the extreme condition acting as a filter or the purpose of human activities, it should be directly based on such general objectives of human systems as energy expenditure, human total effort and financial costs, risks associated with business success, and so on.

Under the conditions of change, these global goals or principles of managing human activities are replaced by lower level practical principles that are direct consequences of global principles, but can also be quantitatively linked to equations of state.

In practice, this means finding an expression for the condition of extreme human activity, consisting of the parameters of the equation of state, to obtain a field of solutions compatible with this equation.

In this sense, the main mechanism for managing changes during human activity is the control of the power $H = N * P$ of the human system.

Such change management is carried out by regulating the number of people N and managing their effectiveness P, not forgetting that the effectiveness of activities, in addition to the skills and productivity of people, includes tools and infrastructure related to the activity.

Using the power of the human system as an extreme condition means that after another change there is a need to adapt people's capabilities to new conditions as quickly as possible by increasing power and reducing risks, or by reducing power and reducing operating costs.

A quick change in the power of the human system can be achieved by making changes in the direction of the gradient of this quantity in the scalar space of the parameters of the equation of state.

This means that the mathematical formulation of the problem of the emergence of people's rational behavior trajectories would be:

$$\left.\begin{aligned} T * N * P &= C \\ \mathrm{Grad}H &= \mathrm{Grad}(N * P) \end{aligned}\right\} \qquad (4)$$

In the particular case, to control human activity under conditions of change, instead of the gradient of the power of the human system, the control can use the gradient of the number of people, a detailed description of which can be found in [3, 4, 5].

Graphical interpretation of equations of state and life trajectories in terms of nonlinear first-order emergence

If we consider the phenomenon of nonlinear emergence of the first category in terms of change management, then it will have the following graphic interpretation.

First, a graphic representation of human activity of a given complexity C will be a hyperbolic surface (Fig. 4).

Each point on that surface will represent some option of the plan for the given activity.

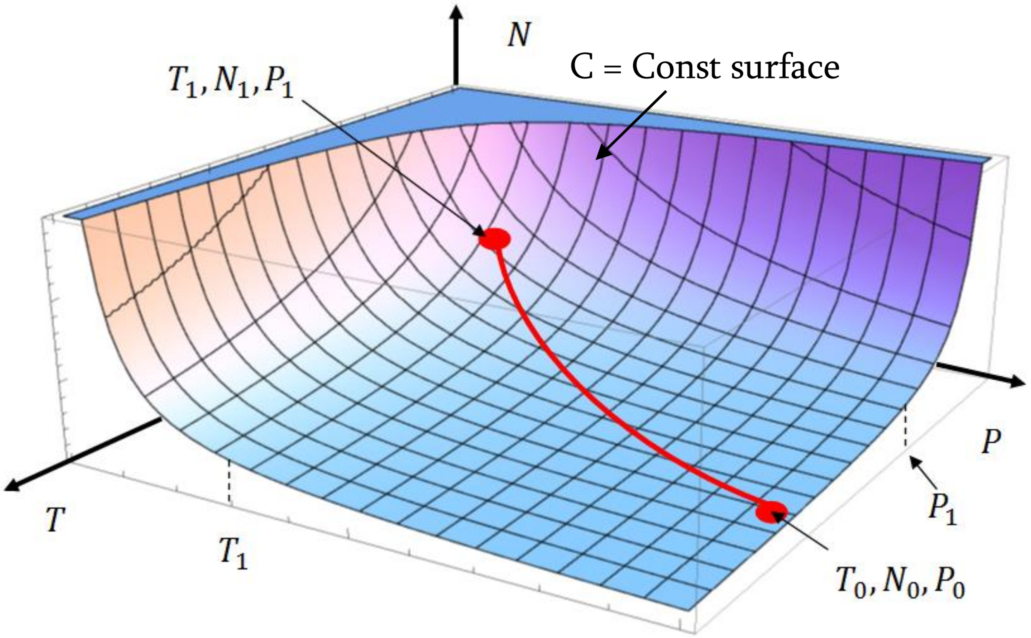

Fig. 6 The hyperbolic surface of the activity of people of complexity C, where the product $T * N * P$ is constant at all its points

Suppose that on the surface (T, N, P) we have a preliminary version of the action plan in the form of a point (T_0, N_0, P_0), which means that N_0 people acting together with efficiency P_0 during the period of time T_0 can carry out an activity with complexity C.

If we want to discuss a new action plan where the duration should be T_1, then this means that the new point of the action plan (T_1, N_1, P_1) should be the intersection point of the plane T_1 = Constant and the red gradient curve of the number of people N.

The projections of the gradient red curve on the $[N, P]$ and $[N, T]$ coordinate planes will be the clear functional dependencies $N = f_1(P)$ and $N = f_2(T)$ respectively.

Thus, the symbiosis and integration of two different qualities, namely, the equation of state and conditions of an extremum of a gradient type, create the necessary mathematical apparatus for controlling human systems under conditions of change and at the same time generate the necessary life trajectories for the system, which is another example of the birth or emergence of a new quality.

The mathematical details related to this problem can be found in [3, 4].

If we consider a more general case of the equation of state, which also takes into account the motivation and errors of people, one can get [8]

$$N * T * M * I_{sMax} = (1 + a_s t_{as}) * W * D, \qquad (5)$$

Even without the use of special mathematical means, it is obvious that equation (5) contains many nonlinear relationships between the parameters of human systems.

For example, it is obvious that the intensities I_{sMax} and a_s are in nonlinear functional relationshipc with the difficulty of the actions D.

In addition, there are an obvious nonlinear relationships between the size W of a human activity and the number of people N and the duration T of activity, etc.

One of the main goals of the mathematical theory of human systems is to identify the aforementioned functional relationships using joint solutions of equations of state and additional conditions.

Nonlinear mathematical models of the emergence of the second category

Consider an example of human activity that shows that the development of nonlinear mathematical models of the second category of emergence phenomena is a critical need.

As we have seen above, any purpose-oriented human system has the upper limit of its capabilities.

The closer the activity of the human system to the upper limit of its capabilities, the more effective the result of its activity in the form of emergence will be in the competitive struggle, but if that activity approaches the upper limit of its potential, its risk of failure will increase dramatically.

In particular, considering a project team as a human system, the presence of the upper limit of that team's capability can significantly influence project planning and implementation.

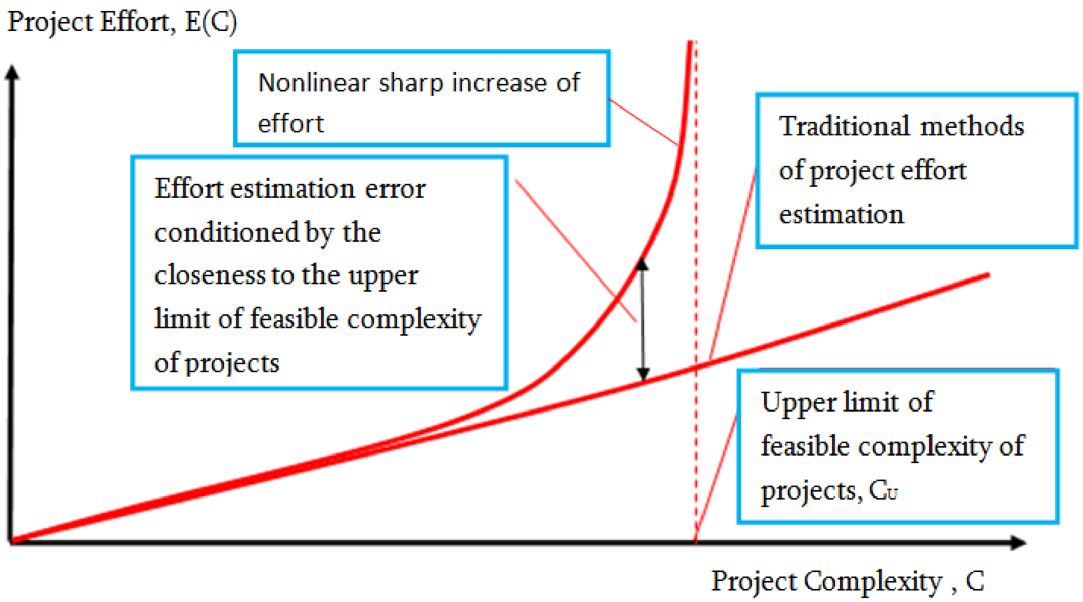

Fig. 7 Close to the upper limits of the capabilities of project teams, the behavior of the total effort required to implement the project is non-linear, the neglect of which can lead to failure of the project
The figure shows the dependence of effort on the complexity of the project using the traditional linear and non-linear approach, which takes into account the upper limit of the feasible complexity of the project.

The importance of this issue is due to the fact that in some areas of human activities the failure rate of projects is very high, resulting in huge financial losses.

One of the main reasons for this phenomenon is insufficient funding due to an incorrect assessment of the efforts required for successful implementation of projects (Fig.7).

Innovative modern projects in most cases are implemented in a competitive mode and, for this reason, are in the upper limit of people's capabilities and project feasibility of increased risks.

Therefore, the effort required to implement such projects should be estimated using non-linear mathematical models.

Otherwise, without taking into account the upper limit of people's capabilities, the project will not be adequately funded, which may become one of the main reasons for its further failure.

This in turn means that the mathematical model of the difficulty of human actions must be refined to solve this problem by the method of state equations.

The peculiarity of this model should be that the real difficulty of human actions D_{Real} should also take into account the distance of difficulty D from the upper limit D_U, that is, the value of $D_U - D$.

Based on these considerations, the mathematical model of the real difficulty of human actions in a simple case will look as follows.

$$D_{Real} = \frac{D_U * \mathrm{D}}{D_U - \mathrm{D}} . \qquad (6)$$

Simple mathematical models of nonlinear emergence

Substituting the value of D_{Real} from expression (6) into the equation of state (1) instead of the difficulty of actions D, and taking into account the number of people N, we obtain a new nonlinear equation of state

$$T = \frac{W}{N * M * P_{Max}} * \frac{D_U * \mathrm{D}}{D_U - \mathrm{D}} \qquad (7)$$

With this equation and the difficulty model of human actions (6), one can modify the linear models given in Part 1 of the article by making them non-linear mathematical models of separation of powers and state structure [8].

In particular, the non-linear mathematical model of the joint activities of the legislative, executive and judicial branches of government will look like this.

$$N_L * T * M_L * P_{sL} = (1 + a_{sL} t_{asL}) * W_L * \frac{D_{UL} * D_L}{D_{UL} - D_L} , \qquad (8\text{-}1)$$

$$N_E * T * M_E * P_{sE} = (1 + a_{sE} t_{asE}) * W_E * \frac{D_{UE} * D_E}{D_{UE} - D_E} , \qquad (8\text{-}2)$$

$$N_J * T * M_J * P_{sJ} = \left(1 + a_{sJ} t_{asJ}\right) * W_J * \frac{D_{UJ} * D_J}{D_{UJ} - D_J} . \qquad (8\text{-}3)$$

In the same way one can obtain the non-linear analogues of all linear models mentioned in Part 1 of this article.

As in the case of a linear emergence, for a nonlinear emergence, functional dependencies between the parameters of the nonlinear equation of state (7) can also be obtained.

For this, it is necessary to jointly solve the equation of state (7) and any objective function related to the activity of people or an extremum condition, which, as in the linear case, can be a condition of the gradient of the number of people $Grad\ N$.

In this case, the problem of the emergence of trajectories of rational behavior of people in conditions of change in the general case will look as follows.

$$\left.\begin{array}{c} N * T * M * I_{sMax} = (1 + a_s t_{as}) * W * \frac{D_{UJ} * D_J}{D_{UJ} - D_J} \\ \\ Grad\ N. \end{array}\right\} \qquad (9)$$

The solutions of the obtained (9) system will consist of a multitude of functional dependencies between the parameters of the equation of state, which are used to evaluate the results of human activity and the emergence of new qualities.

Conclusions

1. Linear and non-linear modes of human activity contain non-linear functional relationships between the parameters that characterize human systems.

2. Therefore, the change of one or more of the system parameters interrelated with one another leads to nonlinear changes in the other system parameters, which is the basis of quantitative change management.

3. On the one hand, the source of nonlinearity of people's behavior may be a linear equation of state itself, besides the more important source of nonlinearity of people's activities may be their physical, mental, organizational, finansial and other limitations.

4. Based on this, non-linearities associated with human activities, including non-linearities characterizing the emergence phenomenon, can be divided into two categories: the first is the internal non-linearities of the linear equation of state, and the second is non-linearities, the source of which is the limited nature of the physical, mental and other capabilities of people.

5. By modifying linear equations of state using a non-linear model of the difficulty of human actions, one can construct non-linear mathematical models for generating the results of human activity and emergence.

6. Non-linear functional relationships characterizing human activities are the results of joint solutions of equations of state and extreme conditions that characterize human behavior and goals.

7. Since the sources of emergence are various modes of human activity, the aforementioned nonlinear functional relationships between the parameters of the equation of state are the basis for mathematical modeling of the emergence phenomenon.

Continuation of work

Mathematical models of nonlinear emergence allow one to study this phenomenon much wider and better than is possible with the help of linear models.

But the phenomenon of emergence itself is very complex and has manifestations that cannot be quantified by even the most accurate non-linear mathematical models.

The reason for this are the random events that accompany the emergence phenomenon, which can be quantified only by the probabilistic methods to which the third part of the article will be devoted.

In addition, it should be noted that the curve shown in Fig. 1 also characterizes a person from a professional point of view, since it reflects both the upper limit of his abilities and the speed of his actions.

This means that this curve can serve as a unique professional passport for people and can be used as a measure of professional level in recruitment.

Using this curve, people's daily lives can also be described in terms of their ability to overcome the difficulties they face in the process of life.

This means that non-linear models of the emergence phenomenon can also serve as a basis for quantitative studies of various social phenomena including the discontent of people, the emergence of new political movements, revolutions, and so on.

References

1. Pavel Barseghyan (2017). **Mathematical Modeling of People's Behavior (Elements of the Mathematical Theory of Human Systems) - 39 pages. (in Russian language).** http://www.iarex.ru/sociology/53498.html

2. Pavel Barseghyan (2017) **“Elements of the Mathematical Theory of Human Systems Part 1: Assessment of the Results of Human Actions and Activities Based on the Method of State Equations ”** *PM World Journal Volume 6, Issue 12 December 2017* – 13 pages. http://pmworldjournal.net/wp-content/uploads/2017/12/pmwj65-Dec2017-Barsegyan-mathematical-equation-of-human-systems-part1-featured-paper.pdf

3. Pavel Barseghyan (2009) **Principles of Top-Down Quantitative Analysis of Projects: Part 2: Analytical Derivation of Functional Relationships between Project Parameters without Project Data.** PM World Today – June 2009 (Vol. XI, Issue VI). 16 pages. http://www.scribd.com/doc/113911821/Principles-of-Top-down-Quantitative-Analysis-of-Projects-Part-2-Analytical-Derivation-of-Functional-Relationships-Between-Project-Parameters-Without-P

4. Pavel Barseghyan (2013) “**Equilibrium and Extreme Principles in Discovering Unknown Relationships from Big Data: Part 2: Non-statistical Mathematical Methods in Project Management**” *PM World Journal Volume 3, Issue 1 January 2014* – 14 pages. http://pmworldjournal.net/wp-content/uploads/2014/01/pmwj18-jan2014-barseghyan-big-data-part2-non-statistical-mathematical-methods-FeaturedPaper.pdf

5. Pavel Barseghyan (2005) **Mathematical Theory of Project Management**. Information Technologies and Management (pp. 21 - 81), (in Armenian language).

6. Pavel Barseghyan (2013) "**The need for New Methodological Approaches for filling gaps in Quantitative Project Management**" *PM World Journal Volume 2, Issue 2 January 2013* – 9 pages. http://pmworldjournal.net/wp-content/uploads/2013/01/PMWJ6-Jan2013-BARSEGHYAN-New-Approaches-for-Quantitative-Project-Management-FeaturedPaper.pdf

7. Pavel Barseghyan (2018) **"Elements of the Mathematical Theory of Human Systems Part 5: Quantitative analysis of the benefits and losses of two human systems in the regime of confrontational equilibrium of their interrelations "** *PM World Journal Volume 7, Issue 4 April 2018* – 13 pages. https://pmworldlibrary.net/wp-content/uploads/2018/04/pmwj69-Apr2018-Barseghyan-elements-of-mathematical-theory-of-human-systems-part5.pdf

8. Pavel Barseghyan (2020) **"** Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems (Part 1: Mathematical Modeling of New Quality Emergence from Interactions of Qualitatively Different Phenomena by the Method of State Equations)**"** *PM World Journal Volume 9, Issue 3 April 2020* – 16 pages, https://pmworldlibrary.net/wp-content/uploads/2020/03/pmwj91-Mar2020-Barsegyan-Quantitative-Models-of-Emergence-part1.pdf.

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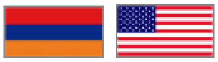

Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia).

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