Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems

Part 3: Probabilistic Mathematical Models of New Quality Emergence[1]

By Pavel Barseghyan, PhD

Abstract

The first two parts of the article were devoted to linear and nonlinear mathematical models of emergense, which describe the generation of human activity results and new qualities at the level of averages of their parameters.

Within the framework of such deterministic approach, many phenomena related to human activity can be described, interpreted and predicted, but these approaches encounter serious difficulties when it is necessary to take into account the various kinds of random events that accompany human activity.

The difficulties associated with such random factors are much greater when performing activities at the upper limits of human abilities.

On the other hand, it is at the upper limits of human abilities and human systems capabilities that the most important events and emergencies related to human progress and safety take place.

An adequate quantitative description of such phenomena is possible only within the framework of probabilistic approaches, which will be discussed in the third part of the article, which includes mathematical models of the emergence at the level of probability distributions of the characteristic parameters of human activity.

Introduction

Although non-linear models of emergence have wide opportunities to adequately reflect different aspects of this important phenomenon, there is one area where they as deterministic models are not capable of describing randomness as an integral part of human activity.

---

[1] How to cite this paper: Barseghyan, P. (2020). Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems - Part 3: Probabilistic Mathematical Models of New Quality Emergence; *PM World Journal*, Vol. IX, Issue V, May.

The thing is that the emergence itself is a highly random phenomenon, especially in the areas where it is really about creating, giving birth, and emerging new phenomena and processes.

Emergence deterministic models describe the phenomena and processes in question at the mean level of the parameters, which does not allow them to form an understanding of the likelihood or risk of successful completion of human activities.

The need for probabilistic approaches in this area is due to the fact that some parameters in the equations of state for describing the emergence phenomenon are of a pronounced random character.

To illustrate the essence of the question more precisely, consider the following simplest equation of state of person's activity [1, 2]

$$T * M * P_{Max} = W * D, \tag{1}$$

This equation describes the realization by a person of an activity of magnitude W with motivation M over a period of time T.

The value P_{Max} in equation (1) is the number of successfully performed actions by a person per unit time, and the value D is the degree of difficulty of the actions performed.

Given that one of the main causes of failures in human activity is a delay in the performance of their actions, which gradually accumulates in order to exceed the time allotted for activities, let's consider the use of equation (1) to analyze the duration of an individual's activity:

$$T = \frac{W*D}{M*P_{Max}} \tag{2}$$

In principle, all four parameters contained in this expression can be random values, so that the duration T of activity will also be random.

Depending on many circumstances, these four random variables may have different distribution functions: symmetric, asymmetric, with a heavy tail, etc.

This means that the distribution of activity duration T will be directly determined by the distributions of these four quantities, which in simple linear cases can be symmetric and have a small dispersion, and in complex nonlinear cases it can be asymmetric and have a large dispersion, and in more complex cases - heavy tail.

Given these circumstances, a methodology based on hypothetical asymmetric triangular or other distributions of T values, which are often used for simulation in the field of project management, needs serious adjustments.

The fact is that in the case of attributing an inadequate hypothetical distribution to the duration T of human activity and building on this a strategy for simulating this activity, it can lead to unreasonable decisions and, as a result, large losses.

In addition, depending on the specific nature of human activity, the distributions of the four indicated values can take various forms, a combination of which, according to expression (2), can generate different distributions for the duration T.

On the other hand, these four quantities are measurable, and therefore it is more correct to accept partial hypotheses about their distributions, and then obtain the distribution of duration T analytically as a combination of the distribution functions of these parameters.

The significance of the above is even more emphasized if we delve into the functional essence of these four parameters, taking into account their dependence on other important factors.

In particular, if we take into account people's errors, then the duration T will be determined by the following expression [1, 2]

$$T = \frac{(1+at_a)*W*D}{M*I_{Max}}. \qquad (3)$$

Here, the intensity of human errors a, the average time to correct one error t_a, and the intensity of actions I_{Max} are themselves random variables with corresponding distributions.

Thus, even in a simple linear case, estimation of the duration of human activity requires consideration of both the internal nonlinearities of the equation of state and the random nature of the parameters.

The question becomes even more complicated when dealing with the phenomena of the second category of emergence, which also takes into account the upper limits of human abilities [3].

The random duration of a person’s activity is the sum of the random durations of his actions; therefore, the estimation of the duration T is directly related to the central limit theorem [4].

The problem of the distribution of the sum of random variables and the duration of human activity

One of the main problems associated with human activities is the estimation of the duration of that activity and its management during the sequence of actions.

In order to correctly assess the time spent by people on activities, it is first necessary to understand the relationship between the duration of actions with human skills and the difficulty of actions.

More specifically, given that the success of an activity is directly related to the duration of individual actions and delays in their implementation, first of all, the role of non-linearities and randomness in the behavior of people in the above delays should be clarified.

According to the central limit theorem, the distribution of the sum of identically distributed random variables with an increase in the number of terms asymptotically approaches the normal distribution if the variances of the terms are finite [4].

Otherwise, when the variances of the terms are not finite, the distribution of their sum asymptotically approaches the so-called stable distributions.

With regard to the duration of people's activities, this means that if for whatever reason people's actions begin to be delayed, their dispersion will increase accordingly, leading to the fact that the duration of the activity will no longer have a normal distribution, but with the increase of dispersion, the distribution will initially have right asymmetry and further increase in scattering will result in heavy tail distributions [5, 6, 7].

In terms of quantitative description of human activity and the phenomenon of emergence, it is important to present in a unified manner the randomness of the difficulty of human actions and the nonlinearity and randomness of human properties by means of mathematical models and equations.

The method of equations of state plays an important central role in this regard, since equations of state represent in a unified way the necessary resources for the activities, abilities and capabilities of people and the results of their activities.

Probabilistic interpretation of linear equations of state of human systems and first-order emergence

In the first and second parts of this paper, linear equations of state of a person and human systems were discussed, on the basis of which quantitative deterministic estimates of human activity can be obtained [8, 9].

As already mentioned, a generalization of such estimates in a probabilistic sense will allow us to take into account the random nature of the parameters in the equations of state.

Doing so will enable the circulation of such important random aspects of human activity as the probability of its successful completion and risks of failure.

In particular, for the duration of human activity, equation (1) was obtained, taking into account the random nature of the parameters of which will serve as the basis for a probabilistic interpretation of the results of human activity and first-order emergences.

The making of human decisions and their implementation in the form of actions occurs under conditions of varying degrees of randomness.

Even if we assume that the activity size W in equation (1) can be considered a nonrandom value, the same cannot be said about the motivation M, productivity P, and difficulty of actions D.

On the one hand, mood, health, motivation, laziness, and other circumstances that characterize a person's skills can be random on the other hand, the environment in which his actions are performed can be random.

The environment itself can include both the physical environment with its climatic and other conditions, and the human environment with its system of interpersonal relationships, with its positive or negative attitude towards a person, contributing to or hindering the implementation of his activities, and so on.

The dispersion of the difficulty of human actions will depend strongly on the nature of the activity.

For example, for conveyor operations, the dispersion of the difficulty of human actions will be so small that parameter D can be considered as a deterministic quantity.

As for the creative activity of people, it will have a relatively large dispersion of the difficulty of human actions.

The probabilistic distribution of the duration of human linear activity

To study the problem of probability distribution function of people's actions and activities, consider the expression (1), which one can present as follows:

$$T = \frac{W}{M} * \frac{D}{P_{Max}} \qquad (4)$$

Considering the case when a person with constant motivation M performs a given activity with size W, the probability distribution of duration T will coincide with the distribution of ratio $\frac{D}{P_{Max}}$.

Both the difficulty D of human actions and the effectiveness P_{Max} of a person's activity have probability distributions that, depending on the different circumstances of a person's life and activities, lie between pure determinism and heavy tail distributions.

It also means that the dispersion value of these distributions in real life is between zero and very large numbers, that is, the distribution of the duration of a person's actions can have an infinite number of different forms and manifestations, which will be the result of an infinite number of combinations of distributions of the quantities D and P_{Max} [5, 6, 7].

Consider, for example, the often-occurring case when the quantities D and P_{Max} have normal distributions.

In this case, the duration of human activity T will have a Cauchy distribution, which is a heavy-tail distribution [5, 7].

This means that even the simplest linear activities of people can have serious delays, which can be the result of increased difficulty in the actions performed and low human skills.

Such delays in human activity may also occur when the difficulty of operations has a heavy- tail distribution that is characteristic of scientific, research and other creative work.

Similar delays are possible in the case of trivial laziness of people and in conflict situations within human groups, when the performance of their activity approaches zero.

The discussed cases are examples of delay emergencies in the linear activity of human beings, which are quantitatively described through probabilistic interpretations of state equations.

In addition, in the framework of linear equations of state, one can consider both ordinary and uniform human life events, and many other social phenomena.

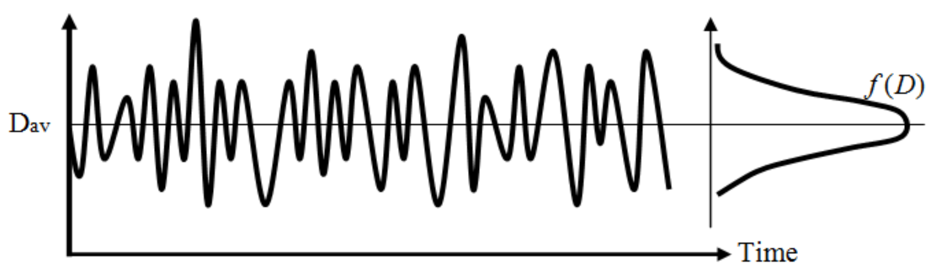


Fig.1. Random process that reflect the difficulty of human work (The difficulties of the serial human actions differ from one another and comprise a random process with some distribution function $f(D)$)

Fig. 1 shows an example of a homogeneous human activity in which the sequence of his random actions is presented as a random process.

The difficulty of these operations is to some extent centered around the mean value D_{Av} and can be approximated by the normal distribution density function $f(D)$.

Fig. 2 illustrates a person's activity process in which he transforms a normal random process of a sequence of relatively simple actions by the linear part of its non-linear characteristic into a random process of the duration of actions, which can again be represented by the density of the normal distribution φ (T).

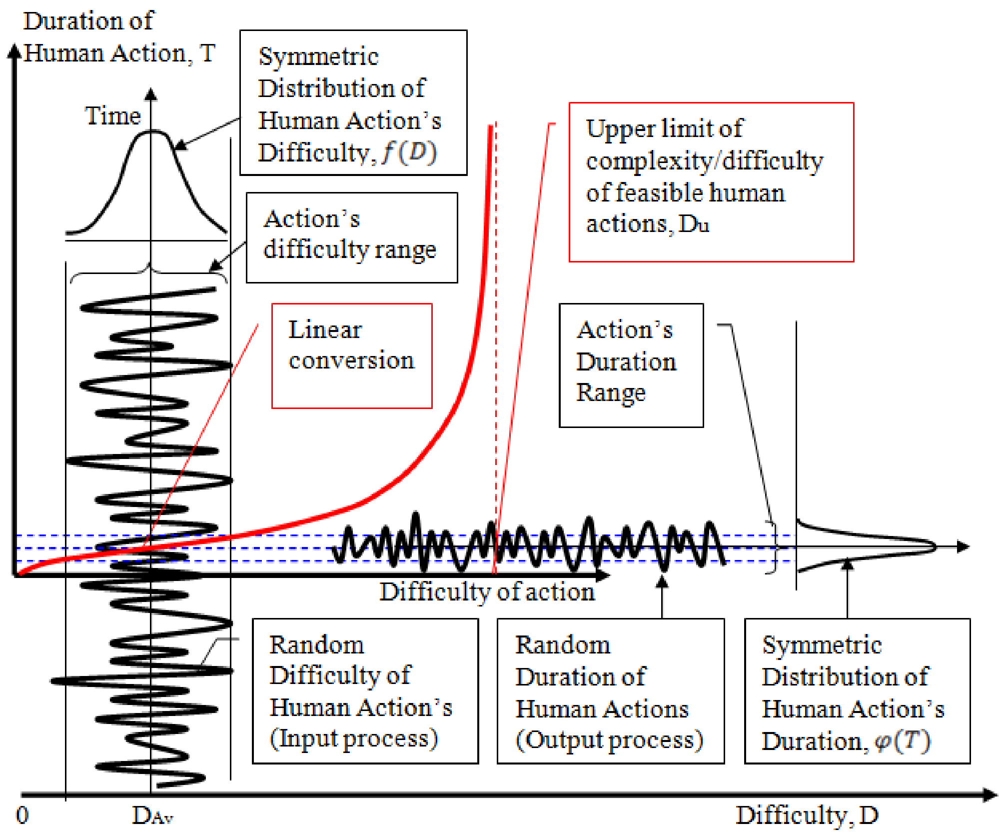


Fig. 2. Linear transformation of difficulty of action into duration of action
(Human as a system transforms the input random process of the difficulty of sequential actions into the output process of the duration of actions)

The two images above represent the probabilistic model of linear emergence, whose mathematical details can be found here [3, 10].

Probabilistic interpretation of nonlinear equations of state and second-order emergence

The non-linear models of emergence, discussed in the second part of this article allow us to introduce the upper boundaries of human capabilities, without which a quantitative representation of the behavior and activities of man and human systems would be inadequate to reality.

But on the other hand, it is impossible to represent probabilistic aspects of this phenomenon, including the probability of its occurrence or the risks of its non-occurrence, by mathematical models of linear and nonlinear emergence of a deterministic nature.

That is, these deterministic models, in turn, have an upper limit of their capabilities, since any estimate made on their basis is valid for the average values of the parameters, since they do not take into account their random nature in the equations of state.

However, if we are interested in the probability of the occurrence phenomenon or the risks of its non-occurrence, then using analytical results representing the deterministic emergence phenomenon and considering them as functions of a random variable, one can obtain analytical estimates of these probabilities [11, 12].

Without going deeper into the mathematical details of the problem, which are partially presented in the above-mentioned articles, let us consider at a qualitative level a few typical examples of probabilistic emergence phenomena.

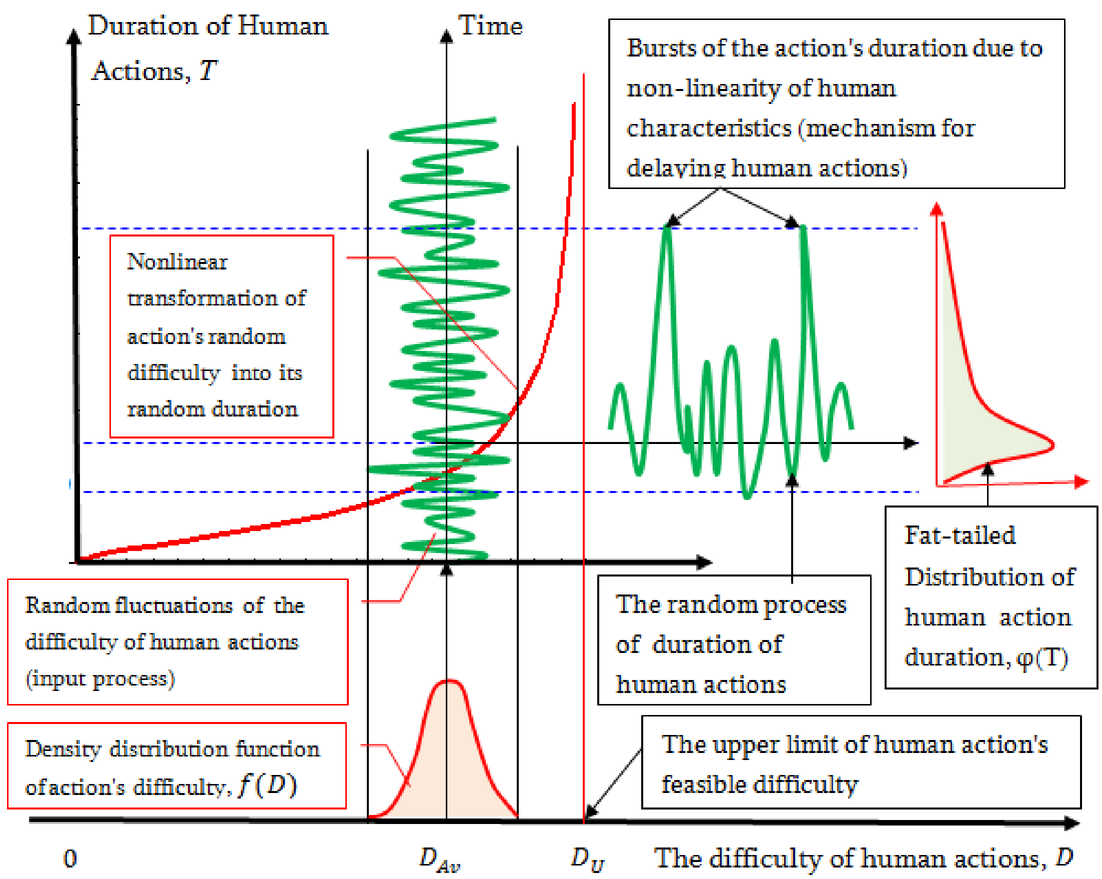


Fig. 3 Probabilistic picture of the execution of actions by a person in the nonlinear field of transformation difficulty - duration

The red curve depicted in Fig. 3 represents a nonlinear dependence of the duration of a person's action on the difficulty of the action.

In the case of the simplest homogeneous human activity, random difficulties in the sequence of his actions can be represented by a vertical green curve and the corresponding probability density function f (D).

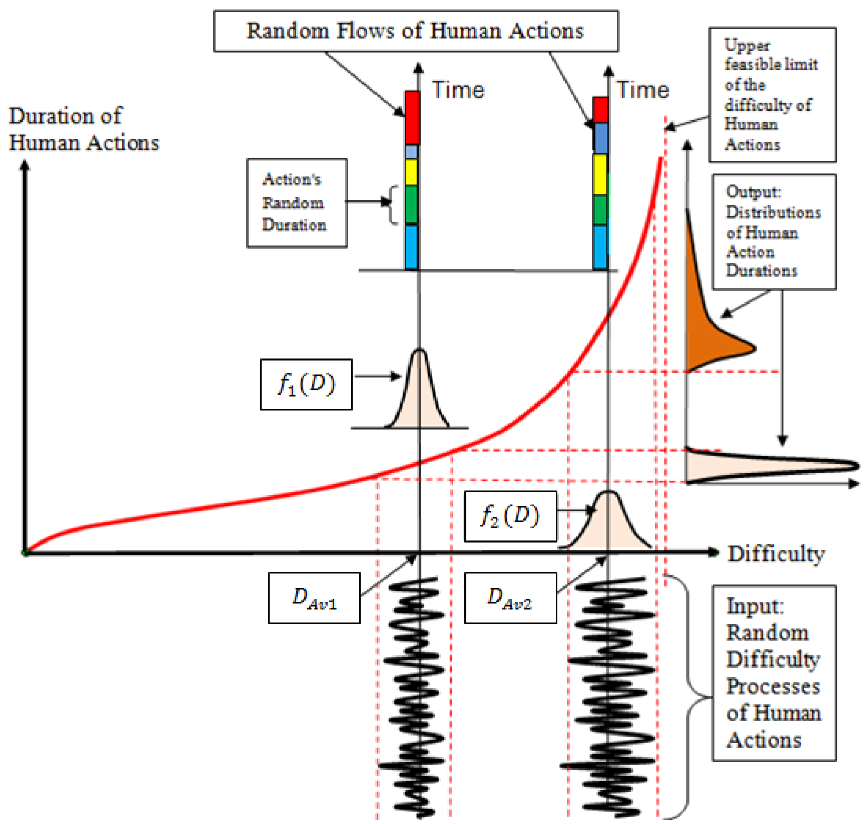


Fig. 4 Joint probabilistic picture of linear and nonlinear human action cases

A human with a non-linear red characteristic transforms an incoming random stream of the difficulties of actions into an output stream of random sequences of action durations, which have a corresponding probability distribution density function $\varphi(T)$

The above is a typical example of a probabilistic analysis of the emergence phenomenon of delays in human activity.

For a more comprehensive consideration of the question one should use the expression of action's duration which also takes into account the D_U upper limit of the action's difficulty [12]:.

$$T = \frac{W}{M*P_{Max}} * \frac{D_U*\mathrm{D}}{D_U-\mathrm{D}} \quad (5)$$

It can be seen from this expression that in the general nonlinear case, the random duration of a person's action is a function of several random variables, including the difficulty of the action D, its upper limit D_U, the effectiveness of P_{Max} and the motivation M.

Fig.4 shows together linear and non-linear cases of analysis of the duration of a person's actions for two random input flows of difficulty of actions.

The output functions of the probability distribution density of the actions shown in the figure serve as a basis for estimating the probability and risk of successful completion of human activity [3].

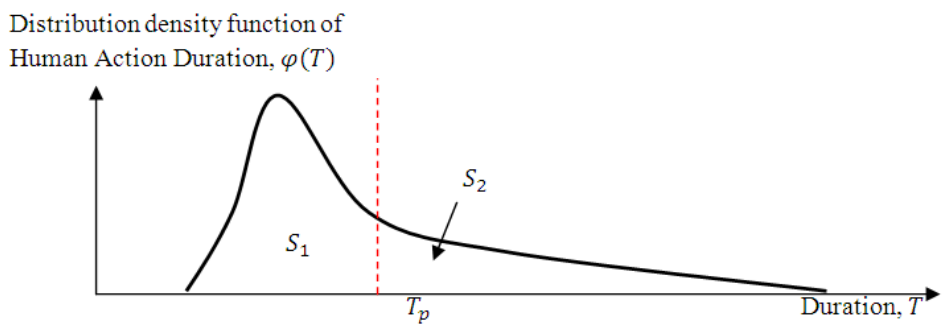


Fig.5. Distribution Function of the human action duration. T_p - is the action's planned time

Fig. 5 shows the probability distribution density function of the action's duration along with the planned duration of the action T_p.

The areas S_1 and S_2 shown in the picture are probabilities of successful and unsuccessful completion of the action that can be easily determined by the density function $\varphi(T)$.

Such quantitative analysis of a person's activities can have many practical applications.

In particular, a probabilistic analysis of human activity can be used to justify the optimal degree of difficulty of tasks that a manager assigns to a person in order to maximize the effectiveness of his activity.

Fig. 6 shows that the extent of the difficulty of the job done by a person to maximize the overall benefit of a person's activity must be close to the optimal value of action's difficulty specific to that person.

The picture shown in Fig. 6 and the optimal level of difficulty in human activity is the result of the overlap of two trends.

The first tendency is that the more complex and more difficult a person's activity is, the higher the risk of its non-fulfillment and the less the expected benefit from it.

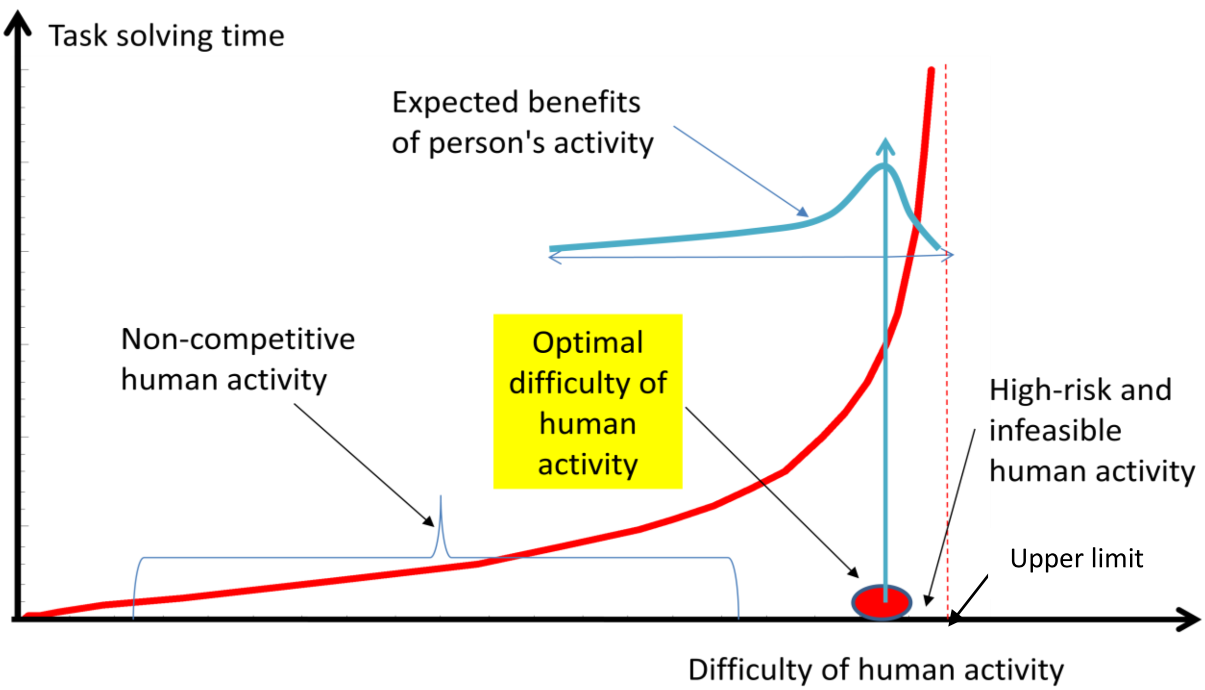


Fig. 6 The expected benefit from human activities directly depends on the degree of difficulty of his actions and has a maximum in the area bordering the upper limit of the difficulty of actions.

The second trend is that the smaller the complexity of human activity, the easier it will be fulfilled, but, on the other hand, the competitiveness of such activities and, consequently, the expected benefits will be lower.

This fact shows how important it is for management to set the right tasks for employees, taking into account the professional and socio-psychological characteristics of people [3].

The above example shows how to maximize the expected benefits of human activities based on a probabilistic analysis of the emergence of a task assignment by managers.

Fig. 6 also shows that before hiring people, their skills and abilities must be clarified in accordance with the curves in this figure.

The above probability analysis of a person's activities can also be applied to various human systems, taking into account the number of people involved in the activity, their error properties and other parameters representing human behavior.

Probabilistic graphical analysis of the emergence of losses of human systems

The method of state equations is universal in nature and allows to quantitatively describe the interaction between human systems in the form of cooperation and confrontation and related emergence phenomena [13, 14, 15].

In particular, the confrontation between human systems is quantitatively described using mathematical models of their mutual pressure on each other.

The activities of the people through which they exert pressure on the opposite side are quantitatively described by equations of state.

The pressure exerted on the human system leads to its losses, which can be material, non-material, financial, etc.

Each human system has an upper limit of resistance to the pressure of external forces upon approaching which its losses increase sharply (Fig.7).

The red curve shown in Fig. 7 is the dependence of the losses of the human system on the pressure exerted on it, which is linear in the case of small values of pressure, and nonlinear in the case of large values.

The figure shows two random pressure processes having average values $R_{Av1}$and R_{Av2}, which act on the same human system in the form of sequences of actions performed on him,

The input processes of two random pressures are respectively described by two probability density functions $f_1(R)$ and $f_2(R)$.

The human system has an upper boundary R_{UL} of resistance to external pressure and a characteristic represented by a red curve that transforms the incoming random pressure process into output loss processes represented by the probability density functions $\varphi_1(L)$ and $\varphi_2(L)$.

Of these, the density function $\varphi_1(L)$ is symmetric as it corresponds to the linear portion of the human system's red characteristic curve, and the density function $\varphi_2(L)$ is asymmetric because it corresponds to the nonlinear portion of the same characteristic curve.

In Fig. 8 shows the loss density function $\varphi(L)$ together with the critical loss valueL_c.

The area S_2 under the curve, located to the right of the critical point of lossL_c, is the probability of failure of the human system due to losses, and the area S_1 is the probability of the relative success of the system.

In fact, Fig. 6 depicts the loss emergence phenomenon of one human system as a result of mutual pressure with another human system.

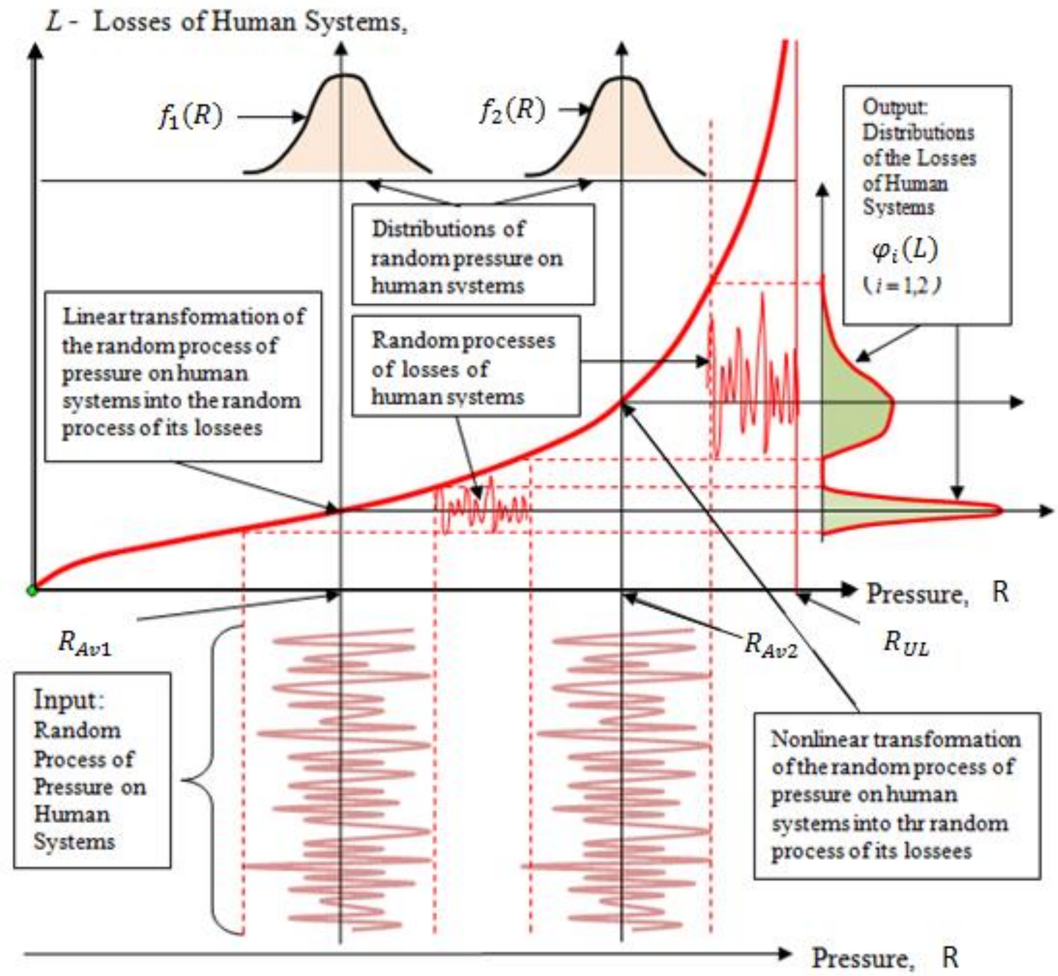


Fig.7 Transformation of two random processes of pressure on the human system into random processes of its losses

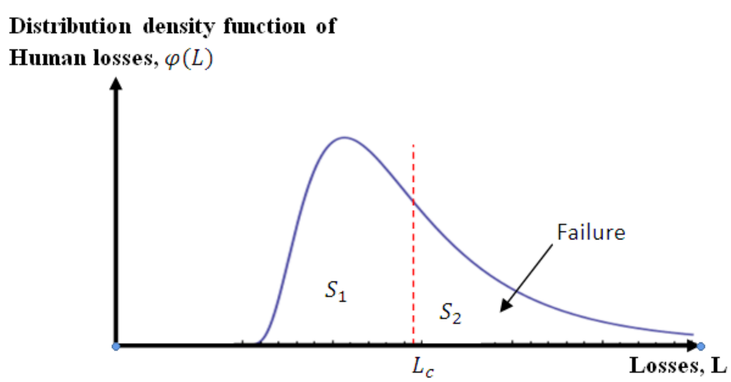


Fig.8.Distribution density function of the human system's losses. L_c - is the critical amount of losses.

Details of mathematical modeling of human system losses based on a non-linear deterministic approach are presented here [15].

Conclusions

1. The method of equation of state of behavior and activity of human systems allows us to obtain the dependence of the duration of action and activity of a person on his skills and the degree of difficulty of activity in an analytical way.

2. Since both people's abilities and the difficulties associated with their activities are random in nature with their probability distribution functions, the duration of their actions and activities will also be probabilistic with its respective distribution functions.

3. Human systems are nonlinear converters that transform random input processes that may have the nature of the difficulty of human actions or pressures on the human system into output random processes.

4. The emergence of delays in human actions and activities can be studied and evaluated through the probabilistic interpretation of equations of state.

5. The pressure of one human system on another human system causes an emergence of its losses, which can also be quantified by means of a probabilistic interpretation of equations of state.

6. A probabilistic analysis of the equations of state of various types of emergences presented in this work shows that other social phenomena can also be studied by the same methods, including the phenomena of mass panic, revolutions, wars, and other phenomena such as "black swans".

Continuation of work

The duration of most actions and activities of people and human systems is so short that their main high-level parameters in the indicated period of time remain constant or undergo minor changes.

In such cases, the behavior and activities of human systems are described by equations of the algebraic nature using deterministic and probabilistic approaches.

However, it also happens that in the course of people's lives and activities, their system parameters undergo significant changes, in which an adequate quantitative description of the behavior and activities of human systems is possible only with the help of dynamic mathematical models, more specifically, with the help of differential equations.

The next part of the work will be devoted to the quantitative study of the phenomenon of dynamic emergence in human systems using differential equations, which, in turn, can be interpreted using deterministic or probabilistic approaches.

References

1. **Pavel Barseghyan (2017) Mathematical Modeling of People's Behavior (Elements of the Mathematical Theory of Human Systems) - 39 pages. (in Russian language).** http://www.iarex.ru/sociology/53498.html

2. Pavel Barseghyan (2017) **"Elements of the Mathematical Theory of Human Systems Part 1: Assessment of the Results of Human Actions and Activities Based on the Method of State Equations "** *PM World Journal Volume 6, Issue 12 December 2017* – 13 pages. http://pmworldjournal.net/wp-content/uploads/2017/12/pmwj65-Dec2017-Barsegyan-mathematical-equation-of-human-systems-part1-featured-paper.pdf

3. Pavel Barseghyan. (2009). **Task Assignment as a Crucial Factor for Project Success (Probabilistic Analysis of Task Assignment)**. PM World Today – April 2009 (Vol XI, Issue I**V**). 15 pages. http://www.scribd.com/doc/114019532/Task-Assignment-as-a-Crucial-Factor-for-Project-Success-Probabilistic-Analysis-of-Task-Assignment

4.Wikipedia. Central limit theorem. https://www.google.com/search?q=Central+limit+theorem&oq=Central+limit+theorem&aqs=chrome..69i57j0l7.3175j1j8&sourceid=chrome&ie=UTF-8

5. Wikipedia. Ratio distribution. https://en.wikipedia.org/wiki/Ratio_distribution

6. Wikipedia. Product distribution. https://en.wikipedia.org/wiki/Product_distribution

7. Wikipedia . Heavy-tailed distribution. https://en.wikipedia.org/wiki/Heavy-tailed_distribution

8. Barseghyan, P. (2020). Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems - Part 1: Mathematical Modeling of New Quality Emergence from Interactions of Qualitatively Different Phenomena by the Method of State Equations; PM World Journal, Vol. IX, Issue III, March. https://pmworldlibrary.net/wp-content/uploads/2020/03/pmwj91-Mar2020-Barsegyan-Quantitative-Models-of-Emergence-part1.pdf.

9. Barseghyan, P. (2020) **.** Quantitative Models of Emergence in Light of Mathematical Theory of Human Systems - Part 2: Nonlinear Mathematical Models of New Quality Emergence; *PM World Journal*, *Volume* IX, *Issue 4 April*, https://pmworldlibrary.net/wp-content/uploads/2020/03/pmwj91-Mar2020-Barsegyan-Quantitative-Models-of-Emergence-part1.pdf

10. Pavel Barseghyan (2009) "**Human Effort Dynamics and Schedule Risk Analysis**". *PM World Today* – March 2009 (Vol XI, Issue III). 13 pages. http://www.scribd.com/doc/114481457/Human-Effort-Dynamics-and-Schedule-Risk-Analysis

11. Pavel Barseghyan (2010) "**Nonlinear and Random Accomplishment Time of Human Actions**". *PM World Today* – March 2010 (Vol XII, Issue III). 16 pages. http://www.scribd.com/doc/114051631/Nonlinear-and-Random-Accomplishment-Time-of-Human-Actions

12. Pavel Barseghyan. (2010). **Probabilistic Analysis of the Duration of Human Actions (Analytical Scheduling Risk Analysis).** PM World Today – April 2010 (Vol XII, Issue I**V**). 16 pages. http://www.scribd.com/doc/114079956/Probabilistic-Analysis-of-the-Duration-of-Human-Actions-Analytical-Scheduling-Risk-Analysis

13. Pavel Barseghyan (2018) **"Elements of the Mathematical Theory of Human Systems Part 3: Mathematics of Cooperation and Confrontation of Human Systems"** *PM World Journal Volume 7, Issue 2 February 2018* – 12 pages. http://pmworldjournal.net/wp-content/uploads/2018/02/pmwj67-Feb2018-Barseghyan-mathematics-of-cooperation-and-confrontation-featured-paper.pdf

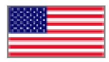
14. Pavel Barseghyan (2018) **"Elements of the Mathematical Theory of Human Systems Part 4: Quantitative interpretation of the victory, defeat and concessions of human systems by the method of state equations "** *PM World Journal Volume 7, Issue 3 March 2018* – 13 pages. https://pmworldjournal.net/wp-content/uploads/2018/03/pmwj68-Mar2018-Barseghyan-mathematical-theory-of-human-systems-part4.pdf

15. Pavel Barseghyan (2018) **"Elements of the Mathematical Theory of Human Systems Part 5: Quantitative analysis of the benefits and losses of two human systems in the regime of confrontational equilibrium of their interrelations "** *PM World Journal Volume 7, Issue 4 April 2018* – 13 pages. https://pmworldlibrary.net/wp-content/uploads/2018/04/pmwj69-Apr2018-Barseghyan-elements-of-mathematical-theory-of-human-systems-part5.pdf

About the Author

Pavel Barseghyan, PhD

Yerevan, Armenia
Plano, Texas, USA

Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia).

Pavel's publications can be found here: http://www.scribd.com/pbarseghyan and here: http://pavelbarseghyan.wordpress.com/. Pavel can be contacted at terbpl@gmail.com