

## **Quantitative analysis of the efficiency and productivity of the activity of human systems**

### **Part 2: Graphical analysis of the nonlinear functional relationship between team productivity and work difficulty <sup>1</sup>**

**Pavel Barseghyan, PhD**

#### **ABSTRACT**

The multi-parameter equations of state of project work contain all possible functional relationships between the system parameters of the project, obtaining which from the equations of state is one of the main tasks of the mathematical theory of project management.

The first part of the work was devoted to the qualitative analysis of the functional relationship between the productivity of the project team and the team size using graphical means.

The second part of the work is devoted to a qualitative analysis of the functional relationship between the productivity of the project team and the complexity or degree of difficulty of the project, taking into account the nonlinearity of this relationship.

The nonlinearity of this functional relationship is explained by the fact that both the members of the project team and the team as a whole have upper feasible limits of difficulties in their work.

#### **INTRODUCTION**

The activities of people involved in the implementation of projects of different scale and content can be viewed as a complex system, the quantitative description of which has the form of multi-parameter equations of state [1, 2].

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As noted in the first part of the article, in the simplest case, the project activities of people can be represented by the following equation of state [3]

$$N * T * P = W * D. \quad (1)$$

The meaning of this equation is that if there is a project of size  $W$  and difficulty  $D$ , then for its implementation it is necessary to have a development team of  $N$  people who, working for a given period  $T$ , with productivity  $P$ , can successfully complete the project.

This is a deterministic interpretation of the meaning of the equation of state (1), which is the basis for understanding, using qualitative and quantitative approaches, the set of functional relationships that exist between the five system parameters of the project included in this equation.

The first part of the article, with qualitative approaches and graphical means, discussed the functional relationship between project team productivity  $P$  and team size  $N$ , for relatively small projects.

This constraint allows, when modeling the complexity of the project, to be satisfied with a simple two-parameter model, namely  $C_d = W * D$ , and a three-parameter model for the complexity  $C_s = N * T * P$  of human activities, which balances the complexity  $C_d$  of the project.

Since all the parameters in equation (1) are connected by functional relationships, it is sufficient that any of these parameters, say, the project size  $W$  or the value  $D$  of its difficulty, were changed for some reason to cause some changes in other parameters of the equation.

Accordingly, since the planning and implementation of projects is carried out under conditions of constant change, the knowledge of the mentioned inter-parametric functional relationships acquires a key meaning and significance in the process of project management.

The productivity of a project team depends not only on the number of people  $N$ , but also on many other factors, including the difficulty of the project work  $D$ , the size of the project  $W$ , the working atmosphere in the project team, the goals of the people, their motivation, the level of their professionalism, and others.

With regard to the equation of state of project works (1) and the equations of state of human systems in general, there are a number of important considerations regarding their use for practical purposes.

1. These equations are not mathematical equations in the usual sense of the word and their use as such without additional conditions can lead to logical contradictions and misinterpretations.

2. To obtain or extract functional relationships between the parameters of the system contained in the equations of state, additional conditions are required in the form of goals of human activity and in the form of restrictions on that activity.

To avoid these difficulties when using equations of state, let us consider several examples of their possible misinterpretations.

### **Examples of possible misinterpretations of the equations of state**

1. Consider the equation of state (1) from the point of view of obtaining from it the functional relationship  $PvsN$ , for which purpose, assume that the other  $T$ ,  $W$  and  $D$  terms of the equation are constant values.

As a result, equation (1) will look like this:

$$P * N = Const, \quad (2)$$

This means that the productivity of the team and the size of the team are in a hyperbolic relationship to each other, which is not true, but this also does not mean that the equation of state in itself is not true.

As shown in the first part of this article, the functional relationship  $PvsN$  has different forms depending on the conditions, and all of them are far from hyperbolic.

In fact, the functional connection  $PvsN$  is indeed contained in the equation of state (1), but it can be obtained only with the help of an additional extreme condition, which is discussed in detail in the author's these works [2, 3, 4].

2. Now consider the functional relationship of  $PvsD$  using the equation of state (1). For this purpose, we assume that the other terms of this equation, besides  $P$  and  $D$ , have constant values.

In this case, for the  $PvsD$  functional connection, one can obtain the following simple linear form

$$P = Const * D, \quad (3)$$

which simply is not true.

In fact, an increase in the difficulty  $D$  in the sphere of an arbitrary human activity necessarily leads to a decrease in the productivity of  $P$ , and as for equality (3), this simply means that to overcome a certain difficulty, people must have the appropriate abilities. in the form of experience, knowledge and skills.

### **Qualitative graphical analysis of the functional dependence $P(D)$ of the performance of human systems on the degree of difficulty of their activities**

It is well known fact that human activity of any complexity is a combination of actions of different scale and difficulty.

Based on this, the description of a person's voluntary activity, whether it is a qualitative representation of it by graphic means or a quantitative description in the form of mathematical equations, leads to an adequate qualitative or quantitative representation of his individual actions.

In this sense, for a qualitative or behavioral representation of a person's actions in the simplest case, the following two functional dependencies can be used:

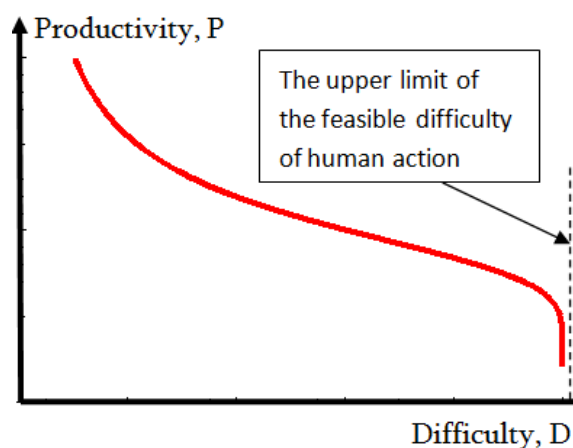


Fig. 1 The effectiveness of human activity is a decreasing function of the difficulty of the activity, especially in the area close to its feasible upper limit, where it has a non-linear sharp decline

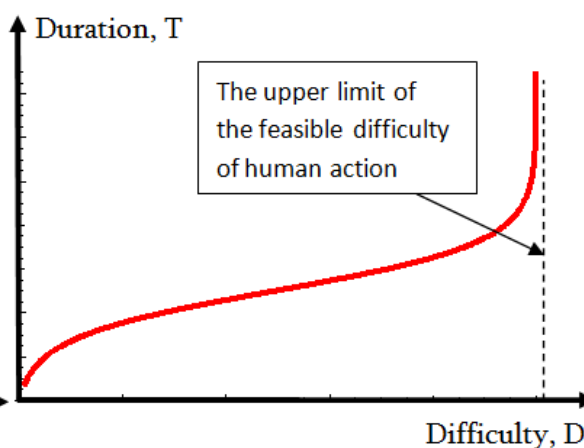


Fig. 2 The duration of human actions is an increasing function of their difficulty, which has a pronounced non-linear character near the feasible upper limit of difficulty

1. Dependence of human performance or the number of successfully completed actions per unit of time  $P$  on the degree of difficulty  $D$  of these actions (Fig. 1).

This functional connection is universal in nature and it adequately describes any human activity, from arbitrary individual actions to activities at the state level.

The peculiarity of any human activity is that in all its manifestations they have an upper limit of feasible difficulty, the approach to which the productivity of their implementation decreases sharply and nonlinearly, and the probability of their successful completion decreases sharply and nonlinearly as well.

This also means that Fig.1 can serve as a basis for a qualitative and quantitative analysis of various risks associated with human activities [5].

2. A qualitative picture of the dependence of the duration of action  $T$  on the degree of its difficulty  $D$  and on the position of its feasible upper limit (Fig. 2).

Both of these curves, which represent different expressions of the same information, have linear segments in the center and non-linear behavior closer to the upper limit of the feasible difficulty of human actions, where majority of decisions related to development and progress, accompanied by significant risks, occur.

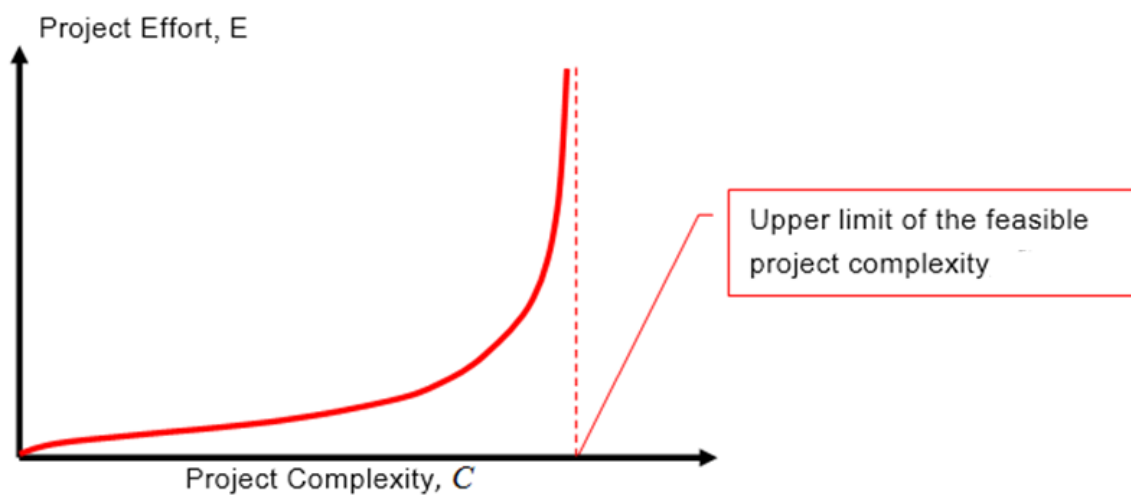


Fig. 3. Modern project management uses close to linear dependences  $E(C)$  to estimate design effort [5], but this is true only when the complexity of the project is quite below the team's capabilities.

This means that the linear parts of the functional dependencies  $P(D)$  and  $T(D)$  can be used in mathematical modeling and assessments of relatively simple human actions, and nonlinear mathematical models can be used in a range close to the upper limit of the feasible difficulty of actions.

Such behavior in people's lives refers not only to individual actions that they perform, but also to activities of any scale, including project-type activities.

The Fig. 3 shows the dependence of the project effort and indirectly also the project cost on the degree of project complexity taking into account the upper limit of the project team's capabilities.

The effort  $E$  required to implement the project is linearly dependent on the complexity of the project  $C$  only if this complexity is small enough compared to the capabilities of the project team.

Otherwise, when the complexity of the project approaches the upper limit of the capabilities of the executing team, the functional relationship  $E(C)$  has a pronounced non-linear character.

In addition, the closer the project's complexity is to the upper limit of the project team's capabilities, the greater the number of design errors and the number of iterations required to complete the project.

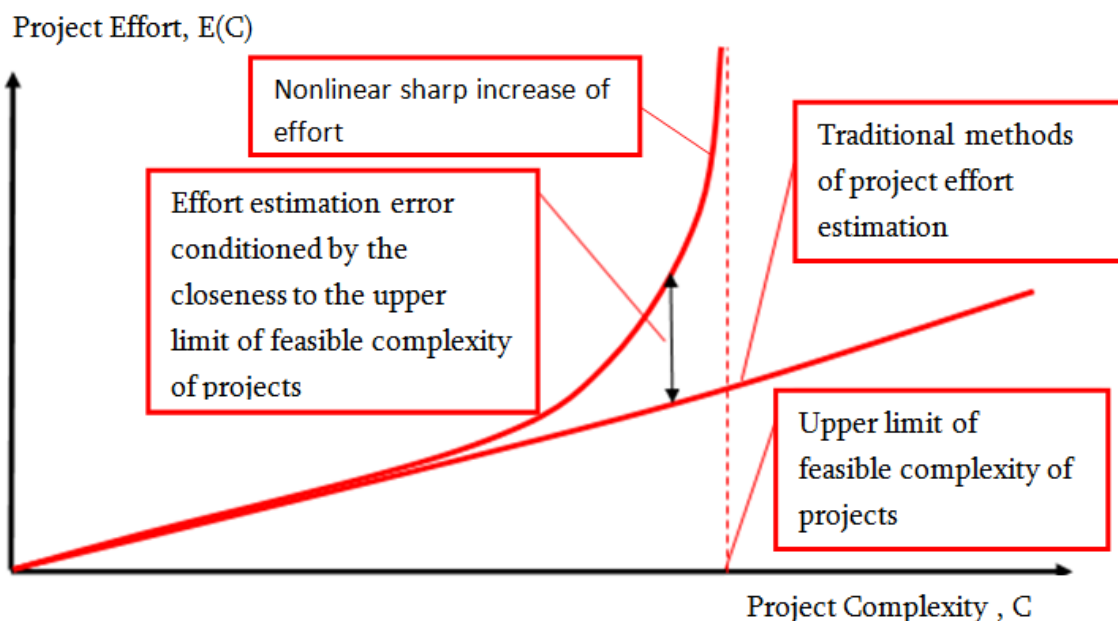


Fig. 4 Failure to take into account the complexity of the anticipated project work and the capabilities of the team performance when evaluating project effort can lead to unexpected negative consequences in the form of project delays and its failure

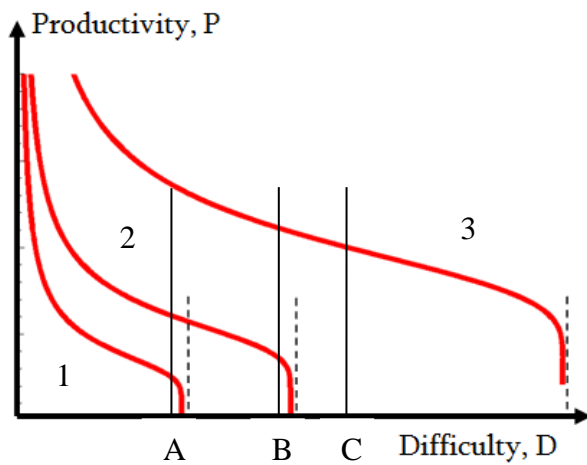


Figure 5 Different people and different project teams have different performance characteristics and upper limits of feasible difficulty

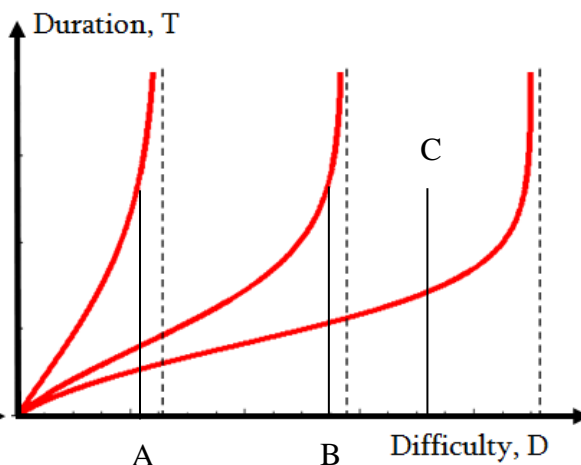


Fig. 6 The duration of the activity of people and human systems depends not only on the scale of this activity, but also on the limits of their capabilities

Therefore, it is very important to take into account the specified non-linear nature of the relationship between project effort and its complexity in order to avoid gross errors in project planning.

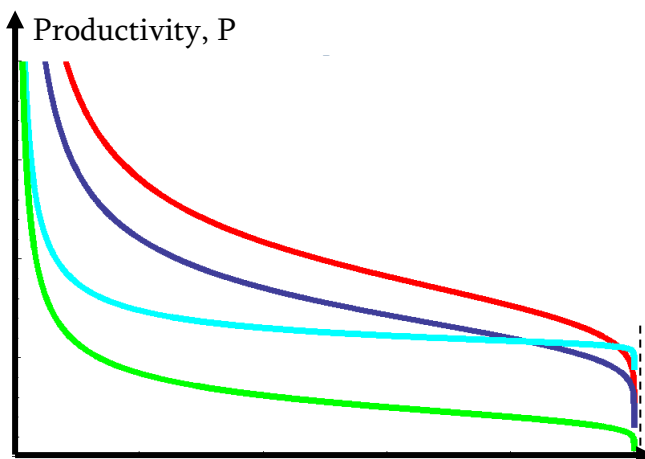


Fig. 7 Four people who have the same upper limit of feasible difficulty of actions, but with different performance characteristics

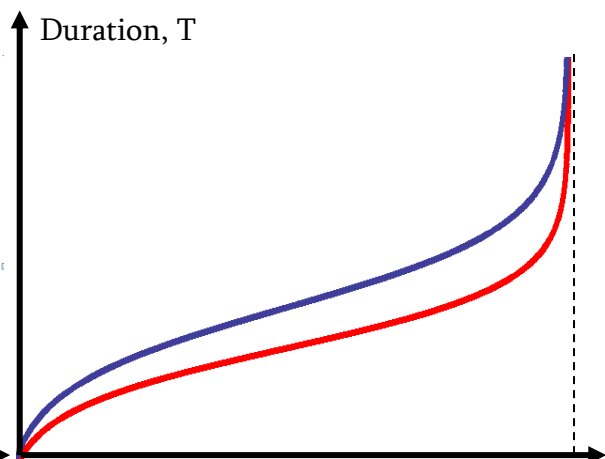


Fig. 8 Two people who have the same limit of feasible difficulty of actions, but with different time characteristics

The fact is that often, not knowing the capabilities of a specific project group and the degree of complexity of the planned project, the cost of the project is estimated or predicted using

close-to-linear models [5], which leads to errors in financing of work and the risk of untimely execution (Fig. 4).

The characteristics  $P(D)$  and  $T(D)$ , representing the executive qualities of people and project teams in the form of families of nonlinear curves, are shown in Fig. 5 and Fig. 6.

A comparative analysis of the families of these curves shows that the work (point A), which is non-linear, complex and practically unrealizable for the team "1", is a linear work of average complexity for the team "2" and an easy linear work for the team "3".

Similarly, the work presented at point "B" is non-linear and unrealizable for team "1", almost unrealizable non-linear difficult work for team "2" and linear work of medium complexity for team "3".

The case when people or project teams have approximately the same upper limit of the feasible difficulty of their actions, but different productivity and different time characteristics, is shown in Figures 7 and 8.

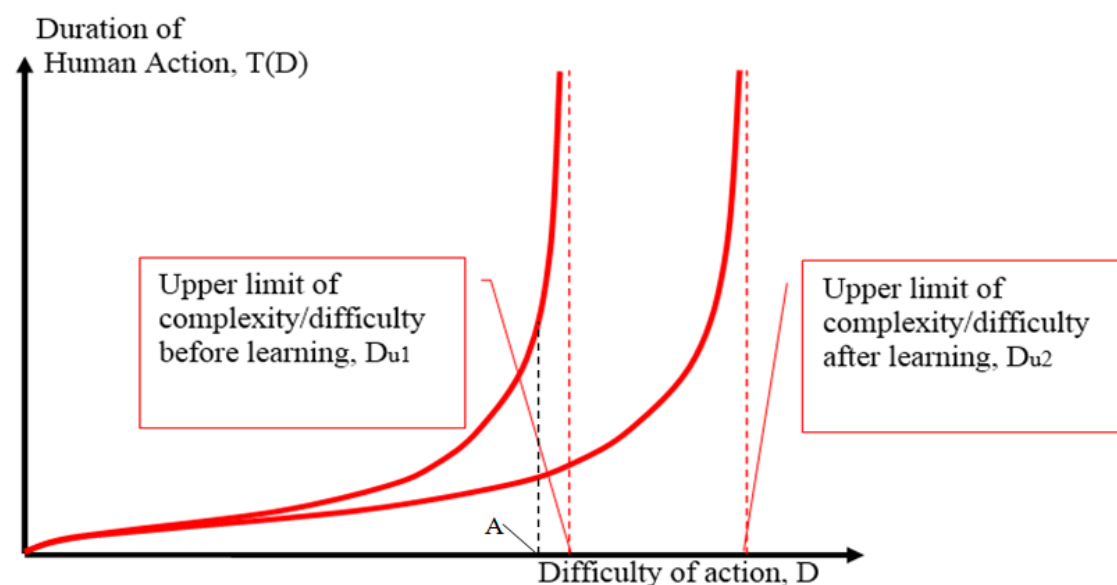


Fig. 9 Curve  $T(D)$  of a person before and after training

The human characteristics  $P(D)$  and  $T(D)$  which represent the working ability of people, project teams, and human systems in general, can also describe the process by which people gain experience, new knowledge and skills in the learning process.



With this approach, the process of acquiring experience, knowledge and skills by people is seen as a shift of the upper limit of their capabilities towards an increase in the complexity of their activities.

In Fig. 9 shown a  $T(D)$  curve representing a person's performance capabilities before and after training, with the result that the difficulty level of the activity represented at point A is non-linear hard work before training, and after training is linear work of average difficulty.

### **Some applications of the curves $P(D)$ and $T(D)$ which characterize the working ability and skills of a person**

#### **1. Assessment of the professional abilities of people**

When hiring people and building project teams, those who do that work in fact measure or identify  $P(D)$  and  $T(D)$  characteristics for individuals and project teams.

Employers are primarily interested in the upper limit of peoples' working abilities.

But a closer look at the problem shows that this value alone is not enough to objectively assess or measure the performance of people.

The curves presented in Fig.7 and Fig. 8 show that people with the same upper limit of ability can vary significantly in speed ( $T(D)$  curve) and error rates in the work flow ( $P(D)$  curve).

In this sense, the characteristics of people  $P(D)$  and  $T(D)$  can have the meaning of their professional passport.

#### **2. The degree of optimal difficulty of the activities of people and human systems**

The fact that people differ in their ability to resist and overcome the difficulties that they face in their activities and, in addition, the problems they face in their activities can have a variety of difficulties, makes the optimal task assignment on the part of management important.

Consider the quantitative meaning of assigning tasks of adequate complexity or difficulty for employees on the part of management using the graph of dependence  $T(D)$ .

The fact is that each member of the development team has an optimal degree of difficulty of the tasks being solved, at which the person's usefulness is, on average, maximum.

This statement makes sense from the point of view of the maximum efficiency of people both at the personal and at the command levels of activity, which has a simple quantitative justification within the framework of the mathematical theory of human systems.

Failure to use a person to the best of his ability is a business loss that requires that he be assigned a job of the greatest possible difficulty.

But, on the one hand, entrusting a person with a more complex and difficult task can make his work more rewarding, but on the other hand, it can increase his risks of not completing the task.

The probabilistic combination of these two tendencies allows us to find the optimal degree of complexity or difficulty of the assigned task, at which the human activity can bring maximum benefit.

This means that misassigning tasks can lead to at least two types of losses.

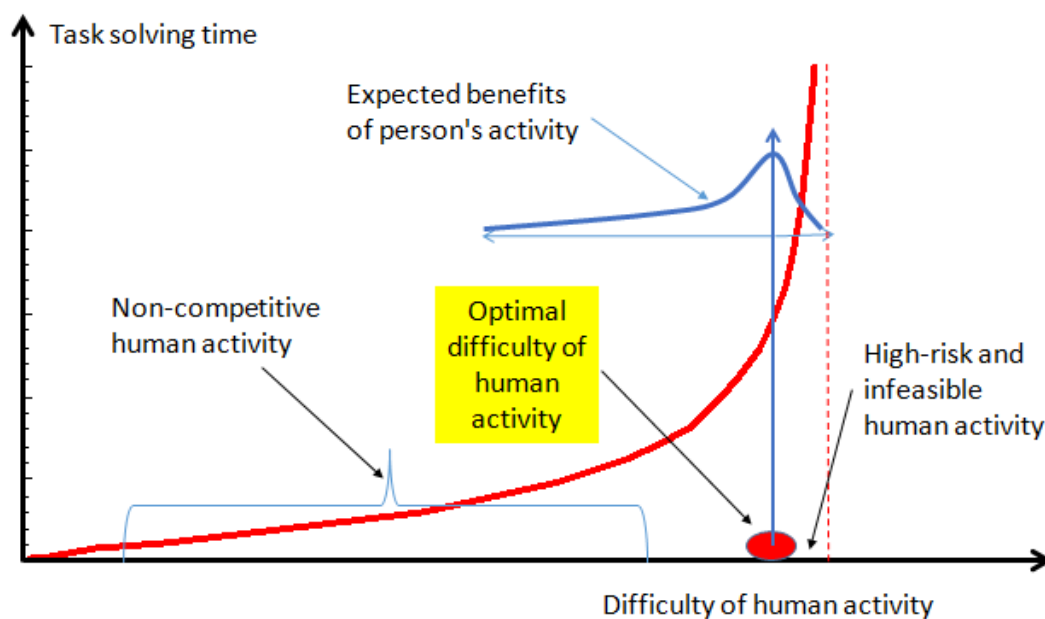


Fig.10: Optimal complexity or difficulty of a human or project team's workload as a trade-off between its complexity and the risk of failure to complete it.

The first is to give a person a simpler task than his abilities, which is a waste in the sense that his capacities are not used to the maximum.

Secondly, giving a person a task is more difficult than he is able to solve, which is a loss in terms of the risk of not fulfilling it.

Consequently, the optimal task that a person is assigned to is a compromise between the complexity or difficulty of the assigned task and the risk of failure to complete it.

A graphical representation of this problem is shown in Fig.10.

## CONCLUSIONS

1. Each person has skills in each specific area of his activity, which can be represented by the characteristics of his performance  $P(D)$  and  $T(D)$ .
2. Each type of human activity has its own degree of difficulty  $D$ , which is the degree of complexity of one unit of his activity.
3. The sum of the difficulties of the actions required to carry out the given activity constitutes the complexity of the entire activity.
4. Any activity performed by a person or a human system has an upper limit of complexity, after which its implementation becomes impossible.
5. The activity of people or human systems near the upper limit of the feasibility of their actions is non-linear, which can lead to delays in these actions, an increase in the number of iterations to complete the actions and the failure of all activities.
6. Most of the activities of an ordinary nature occur in the simple linear parts of the  $P(D)$  and  $T(D)$  characteristics of people, and most of their actions that determine the progress of humanity, mainly occur in the non-linear parts of these characteristics.
7. To determine the feasibility of a particular type of human activity, the problem should be viewed as a result of the collision or interaction of the characteristics of people's skills with the difficulty of their activities.
8. The same project activity, which is non-linear for a team with less experience, knowledge and skills and is difficult to implement, for a team with more experience, knowledge and skills can be considered easy and simple linear work.
9. Currently, the field of project management with its approaches and ideology is mainly in the field of linear or almost linear thinking or paradigm.
10. There are a large number of leading projects in all spheres of human activity, which are inherently deeply non-linear, and therefore risky, for the solution of which linear approaches are not suitable.

11. In the field of management of leading human activities, including the management of various projects that ensure the progress of mankind, it is necessary to move from the paradigm of linear approaches to a non-linear paradigm.

12. Moreover, here we mean a double paradigm shift: in the framework of the deterministic approach to the problem of project estimates, this is a transition from the linear to the nonlinear paradigm, and in the framework of the probabilistic approach, that is a transition from the Gaussian paradigm to the Paretian paradigm.

13. Depending on the professional qualities of people, there is an optimal level of difficulty in their activities, at which the expected result of the work of people will be maximum.

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## About the Author



### **Pavel Barseghyan, PhD**

Yerevan, Armenia  
Plano, Texas, USA



**Dr. Pavel Barseghyan** is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia).

Pavel's publications can be found here: <http://www.scribd.com/pbarseghyan> and here: <http://pavelbarseghyan.wordpress.com/>. Pavel can be contacted at [terbpl@gmail.com](mailto:terbpl@gmail.com)