Quantitative description of the dynamics of interactions of living systems by the method of state equations

Part 1: Derivation of the equations of the predator-prey model from the equations of state of living systems[1]

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Abstract

The mathematical theory of human systems based on equations of state is also applicable for the model representation of the interaction dynamics of living systems, including a quantitative description of the life cycle of animals and their interactions.

The reason for this is the fact that both the life of humans and animals is a sequence of various actions that can be described by equations of state.

This also means that the universal method of equations of state must be in agreement with other known quantitative models representing the life and interactions of living systems.

The only way to prove the validity of such a statement is to derive the known quantitative models of living systems and their interactions basing on the method of state equations.

In this paper, it is shown that the behavior of the predator-prey system can be described by the method of equations of state, and based on this, one can easily obtain the differential equations of the famous predator-prey model of Lotka-Volterra in an analytical way.

This means that in the same way it is possible to obtain other known quantitative laws representing the behavior of living systems.

Moreover, such an approach also can make it possible to obtain analytically new patterns representing the possible manifestations of the behavior of living systems.

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Introduction

All living systems, from the simplest organisms to people, society and civilization, in each situation are guided by the principle of ensuring the continuity and longevity of their life [1].

Since life at any level survives under conditions of various kinds of changes, in the event of an arbitrary change, the biosystem seeks to ensure the continuity of its life with maximum probability.

The course of life of an arbitrary biosystem is a sequence of actions, which is the response of this biosystem to the successive demands of life.

If the course of the life of a biosystem is stable and continuous, then there is a certain balance between the sequence of life requirements and the responses to these requirements on the part of the biosystem.

The concept of such a balance allows for a quantitative interpretation of the principle of continuity and longevity of life in the form of mathematical equations [1].

On the other hand, a quantitative interpretation of this principle is possible only if both the requirements of life and the responses of the biosystem to these requirements can be presented in a parametric form [2].

According to this approach, the requirements of life in an arbitrary period T have the magnitude C_d, which is defined as the product of the size of the requirement of life W by the level of its difficulty D.

$$C_d = W * D \qquad (1)$$

The response of the biosystem to the requests of life has the size C_s, which is determined by the following simple expression

$$C_s = N * T * P, \qquad (2)$$

where N is the number of organisms in a biosystem or the number of people in a social system, T is the duration of processes, P is the productivity of a biosystem for the production of some result.

In the case when there is an equivalence or, ideally, equality of the values of C_d and C_s, the result is the following equation of state of the biosystem or the human system.

$$N * T * P = W * D \qquad (3)$$

This equation is universal in nature and represents the transformation of the energy $N * T * P$ of people or animals into the action or work $W * D$.

In addition, the equation of state (3) has an important feature that the value of W, determined by the current activity of any biosystem, can be considered as a source of generation of results or values necessary to ensure the continuity and longevity of life.

This means that, considering the value of W as a source of the results R of the life cycle of biosystems or human activities, in the first approximation it can be quantitatively represented in a linear form,

$$R = k_R * W, \qquad (4)$$

where the coefficient k_R is the result or value created by the unit W in accordance with the pursued goal and which can sometimes exceed the expectations of the biosystem or the human system.

Substituting the value of W from expression (4), into equation (3) one can obtain a new equation of state,

$$k_R * N * T * P = R * D, \qquad (5)$$

the meaning of which lies in the fact that overcoming the difficulties D in the direction of the desired goal due to the energy and skills of the biosystem, a certain result R is generated.

That is, the equation of state (5) reflects what constitutes the meaning of life and actions of any living system - using various resources and own skills, strive for the maximum result in the direction of the goal, thereby ensuring continuity and longevity of life.

In this sense, the equation of state (5) has a universal character, on the basis of which it is possible to describe and explain the manifestations of behavior and the course of life of any biological and human systems.

For this purpose, let's choose one of the well-known mathematical models of biosystems and, using this model to try to understand the potential capabilities of the equation of state

(5) in the direction of an adequate representation of the behavior of an arbitrary biological or human system.

Having that goal, let's consider the well-known predator-prey model of Lotka-Volterra in the form of a system of differential equations, in order to obtain these well-known equations from the equation of state (5) in an analytical way.

Model Predator - Prey Lotka - Volterra

Let us first briefly introduce the differential equations of this model so that they can be obtained later using a dynamic version of equation (5).

This model is a system of differential equations for the sizes of the populations of prey N and predators M [2].

$$\frac{dN}{dt} = N(a - bM) \qquad (6\text{-}1)$$

$$\frac{dM}{dt} = M(cN - d), \qquad (6\text{-}2)$$

where a, b, c and d are positive constants.

These equations are based on two principles: one relates to the dynamics of the number of prey, and the other to the dynamics of the number of predators.

The first principle states that in the absence of predators and the presence of food, the number of prey will increase exponentially at a rate equal to:

$$\frac{dN}{dt} = a * N \qquad (7\text{-}1)$$

In the presence of predators, the growth rate of prey a will decrease by the value bM and the dynamics of their number will be described by equation (6-1).

The second principle states that in the absence of prey, the number of predators will exponentially decrease at a rate equal to

$$\frac{dM}{dt} = -d * M, \qquad (7\text{-}2)$$

In the presence of prey or food, the rate of decrease in the number of predators d will change by cN, and the dynamics of the number of predators will be described by equation (6-2).

The purpose of this paper is to show that the equations of the predator-prey model (6-1) և (6-2) can be derived from a more general equation based on the method of equations of state, and more specifically using the dynamic equivalent of equation (5).

Static and dynamic equations of state of living systems

In cases where the main system level parameters of a biosystem or a human system do not depend on time, the behavior of systems in a finite time interval T can be described by static equations of state.

Equations (4) and (5) are typical static equations of state.

In cases where the values of the parameters of the system depend on time, the static equations can be valid only for a small interval of time Δt, during which the system can produce some result ΔR.

In this case, equation (5) will look like this:

$$k_R * N * \Delta t * P = \Delta R * D, \qquad (8)$$

which is equivalent to the following differential equation

$$\frac{dR}{dt} = \frac{k_R * N * P}{D} \qquad (9)$$

This dynamic equation of state will be used below for the analytical derivation of the differential equations of the predator-prey model of Lotka-Volterra.

Some peculiarities of the generation or production of results by biosystems

To be able to use the differential equation of state (9) for the analytical derivation of the predator-prey model, let's consider some of the peculiarities of the generation or production of results of actions by living systems.

Each time interval of a person's life and activity gives a certain result, if a person works, he generates a variety of products, if he sleeps, he rests and restores strength, eats food, restores spent energy, and so on.

As far as animals are concerned, their actions 1. produce or generate safety for themselves and danger to their enemies and victims, 2. eat to restore energy and ensure the normal course of life, and, finally, 3. reproduce their own kind to ensure continuity of life.

For the purposes of this article, we are interested in the dynamics of reproduction of predators and prey.

If we look at equation (8) from this point of view, it simply means that N animals reproduce their own kind in the period Δt, that is, for this particular case, the value of the result ΔR should be replaced by the value of ΔN.

This means that the equation of state (9) will have the form

$$\frac{dN}{dt} = \frac{k_R * N * P}{D} \qquad (10)$$

Another important peculiarity is that the productivity P in this equation has the meaning of the rate of change in the number of animals and which is the difference between the rates of their birth P_B and the rate of death P_D.

$$P = P_B - P_D \qquad (11)$$

In general, the performance P can have many different meanings, which correspond to the nature and purpose of the actions of animals or activities performed by people.

Thus, the equation of state for animal reproduction will be as follows:

$$\frac{dN}{dt} = \frac{k_R * N * (P_B - P_D)}{D} \qquad (12)$$

In continuation, let us consider separately the dynamics of the number of prey and the number of predators using equation (12).

Special attention should be paid to the fact that both the analogue of equation (6-1), which represents the dynamics of the number of prey, and the analogue of equation (6-2), which represents the dynamics of the number of predators, will be obtained analytically from the same equation (12).

Differential equation of the number of prey

Equation (12) for the number of prey N will have the form

$$\frac{dN}{dt} = \frac{k_{RN}*N*(P_{BN}-P_{DN})}{D_N}. \qquad (13)$$

The rate of death of prey P_{DN} in this equation is related to the number M of predators, which in the simplest case can be approximated by a linear law

$$P_{DN} = g * M, \qquad (14)$$

where g is the coefficient of proportionality.

Taking into account expression (14), the final form of equation (13) for the dynamics of the number of prey will look as follows

$$\frac{dN}{dt} = \frac{k_{RN}*N}{D_N} * (P_{BN} - g * M): \qquad (15)$$

Differential equation for the number of predators

The dynamics of the number of predators according to equation (12) will look like this:

$$\frac{dM}{dt} = \frac{k_{RM}*M*(P_{BM}-P_{DM})}{D_M} \qquad (16)$$

The intensity P_{BM} of the birth of predators in this equation depends on the number of prey N, which are food for predators and that relationship in the linear approximation will look like this

$$P_{BM} = q * N, \qquad (17)$$

where q is the coefficient of proportionality.

Hence, the final form of the differential equation for the number of predators will be

$$\frac{dM}{dt} = \frac{k_{RM}*M}{D_M} * (q * N - P_{DM}) \qquad (18)$$

Combining the differential equations for the number of prey and predators (16) and (18) into one system of equations, we obtain the predator-prey model, which was derived in a fundamentally new way based on the method of equations of state for living systems.

$$\frac{dN}{dt} = \frac{k_{RN}*N}{D_N} * (P_{BN} - g * M) \qquad (19\text{-}1)$$

$$\frac{dM}{dt} = \frac{k_{RM}*M}{D_M} * (q * N - P_{DM}). \qquad (19\text{-}2)$$

Comparing the obtained equations (19-1) and (19-2) with the equations of the Lotka-Volterra predator-prey model (6-1) and (6-2), one can make sure that these systems of equations are identical in the mathematical and structural sense.

Although the system of equations of the Lotka-Voltaire predator-prey model is based on ecological principles, and equations (19-1) and (19-2) are derived from a much more comprehensive principle, namely the continuity and longevity of the life of living systems, nevertheless they gave the same result, which is very important for further expanding the possibilities and applications of the method of equations of state for analyzing the interactions of living systems.

If we compare the axiomatic powers of the principles underlying these two approaches, we will see that while the principles underlying the Lotka-Volterra model are of a narrow ecological nature, the principle of the continuity of life of living systems underlying the equations of state is applicable to quantitative description of arbitrary systems of animals and human systems.

This circumstance will help to cover the needs of mathematical modeling of the complex problems of the dynamics of interaction of human systems and multidimensional environmental problems with the help of branching the equations of state.

Conclusions

1. Since the equations of state of biosystems and human systems are a direct consequence of the principle of the continuity and longevity of life, and all actions of animals and human activities pursue the goal of ensuring the continuity of life, the equations of state can, in principle, describe, explain and predict the actions of animals and human activities in general.

2. In particular, a quantitative picture of such a complex phenomenon as the coexistence of predators and prey, with its nonlinearity and periodic nature, can be analytically obtained from the dynamic version of the equations of state of this phenomenon.

3. This also means that the method of equations of state of living systems with its static and dynamic approaches can be used in the field of mathematical ecology.

4. In this paper, the classical version of the predator-prey model in the form of the Lotka-Volterra equations was used to illustrate the above, but it is clear that the same approach can be applied to all subsequent versions of this model.

5. Moreover, the present study shows that the method of equations of state can be used to quantitatively describe the dynamics of interactions of a wider class of biosystems, to which the second part of the work will be devoted.

Continuation of research

The equations of state of living systems and human systems are the balance conditions for the actions of animals and the activities of people and, therefore, are universal in nature.

This means that, potentially, using static and dynamic versions of the equations of state, it is possible to quantitatively represent various phenomena and processes accompanying the life of biosystems, to give their qualitative explanations, as well as to plan and predict the actions and activities of human systems.

In particular, it is very promising to take into account the upper limits of the realizable difficulties in the same predator-prey model, which are a source of nonlinearities of a qualitatively new nature.

In addition, using the method of equations of state, it is possible to quantitatively describe the case when two predators are also preys of each other.

If we consider this case in a broader context, then it can be used to model the behavior of human systems, which are simultaneously in a state of confrontation and cooperation.

In this sense, the mathematical modeling of the coexistence of two or three superpowers by the method of equations of state is of great interest.

In addition, mathematical models of various types of social and political processes and phenomena of a competitive type can be constructed by analogy with the predator-prey model.

In this sense, the quantitative representation of the dynamics of civilization as a process of interaction between the gap in human productivity and investments for its elimination, where the productivity gap plays the role of the prey, and the investment plays the role of the predator, is of great practical interest.

References

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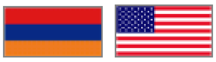

Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia).

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