Quantitative description of the dynamics of interactions of living systems by the method of state equations

Part 2: Productivity Gap vs. Investments: Quantitative analysis based on predator-prey model and state equations[1]

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Abstract

In the process of development, human activity moves from one paradigm of its implementation to another, more effective in terms of human productivity.

In this sense, in the process of development, each paradigm is characterized by its own potential capabilities, which are gradually being exhausted, and this leads to the fact that a dynamic gap between the needs of life and the capabilities of people is gradually growing, the bridging of which requires new investments.

In this sense, the normal development process is the sum of interconnected processes of periodic changes in the dynamic gap in human productivity and investments aimed at bridging this gap.

This article is devoted to the construction of mathematical models of the above-mentioned periodic processes using two different approaches.

The first approach looks at productivity gaps and investment processes with the aim of closing them in terms of the well-known predator-prey model.

The second approach to the same problem is based on the method of dynamic equations of state, the resulting model of which coincides with the results of the first approach, which indicates the great possibilities of practical applications of the method of equations of state.

Introduction

One of the main trends in human history and the development of society is a systematic increase in human labor productivity, based on targeted investments of society.

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Progress in any area of human endeavor is usually accompanied by a range of financial, governmental and geopolitical considerations aimed at achieving or maximizing an overall competitive advantage or generalized benefit.

Here, the generalized benefit is understood as the entire spectrum of material and non-material interests of people, their various combinations depending on the value system of a particular human or social unit.

Often, depending on the circumstances, the desire of people to maximize the generalized gain is replaced by their desire to minimize the generalized loss, which is more applicable to processes of a regression type.

Any progress is based on the dissatisfaction of people or society with the current situation, which is measured by the difference between the size of society's demand and the real capabilities of the human system that implements it.

If, along with this discontent, society has the opportunity to overcome it in the form of the resources and abilities of people, then together with a certain ideology with a tendency to progress, all this can serve as a basis for investing in a specific direction.

From this point of view, the degree of dissatisfaction mentioned above can be expressed as the difference between the desired or potential performance of people in a particular area and their actual performance, which is called the productivity gap [1].

Thus, the dynamics of the development of any sphere of human activity can be represented as a process of interaction and mutual conditionality of the gap in productivity and investments aimed at eliminating it.

A qualitative picture of this process can be presented in the following simplified form.

After another investment, which gradually creates a new generation of tools with higher productivity than the previous generation, and which gradually increases the efficiency of human activities or productivity in a narrow sense, brings with it new thinking and a new wave of innovations that acts as a platform for the next stages of the development process.

Thus, each investment has a certain potential to increase people's productivity, which is spent over time, but, on the other hand, new development opportunities opened up by the same investments create a new, higher level of demand in society that cannot be satisfied within the existing paradigm of productivity.

In other words, each subsequent wave of investment has two types of qualitative consequences, the first of which is a certain potential for productivity growth, which is gradually depleted, and the second is new development opportunities that require a new, higher level of productivity for their effective use.

Since these opportunities offer the promise of greater generalized benefits to society, this paves the way for new, larger investments that begin a new cycle of investment and productivity growth.

If we approach the problem of investment and productivity growth of human activity from a quantitative point of view, one can see that investment and productivity growth are interdependent periodic or quasiperiodic processes with a phase shifted relative to each other.

In this sense, the problem "Productivity gap vs. Investments" is essentially similar to the predator-prey problem and can be quantitatively described using the same mathematical model as the dynamics of the numbers of predators and prey.

In addition, since Productivity gap and Investments are inherently the results of human activities, also they can be quantified by the method of state equations [2, 3].

The further presentation of the article is divided into two parts, the first of which is devoted to the derivation of differential equations for productivity gap and investments, based on local hypotheses, and the second part is to the derivation of similar equations by the method of equations of state.

Productivity Gap vs. Investments: Building a Mathematical Model Based on Hypotheses

With this approach, the construction of a mathematical model of the functional relationship between the productivity gap and investments is based on two hypotheses, the first of which relates to the productivity gap, and the second to investments.

Hypothesis 1. If there is no investment for a long time and the public or industrial demand for productivity increases, then the productivity gap G will also grow rapidly, which can be approximated by an exponential law that satisfies the following differential equation

$$\frac{dG}{dt} = a * G, \qquad (1)$$

where coefficient a is the growth rate of the productivity gap.

To make the meaning of this equation more obvious, note the fact that the larger the performance gap G, the greater the associated potential losses L.

Since the increase in losses becomes sharper after a certain value of the gap in productivity, then the functional dependence $L(G)$ can, in the simplest case, be approximated using a quadratic law.

$$L = c * G^2, \qquad (2)$$

where c is a coefficient associated with a specific area of human activity.

Taking into account expression (2), one can obtain the following differential equation for losses L associated with a productivity gap G

$$\frac{dL}{dt} = 2a * c^{0.5} * L^{0.5} . \qquad (3)$$

Hypothesis 2. If the productivity gap is small and the public or industrial demand for productivity does not increase, then investment in this direction will rapidly decline, which can be approximated by an exponential decay law that will satisfy the following differential equation.

$$\frac{dI}{dt} = -b * I, \qquad (4)$$

where coefficient b is the rate of decline in investments.

The main way to avoid the losses associated with the growth of the productivity gap is to make new investments to increase the productivity of people and tools that will reduce the growth rate of the gap a by Δa, which will lead to the equation

$$\frac{dG}{dt} = (a - \Delta a) * G, \qquad (5)$$

In turn, the value of Δa decrease in the value of a will directly depend on the size of the investment I, which for a linear approximation will look like this:

$$\Delta a \; = \nu * I \qquad (6)$$

where ν is the coefficient of proportionality.

Taking into account expression (6), for the productivity gap G and associated losses L, one can obtain the following differential equations

$$\frac{dG}{dt} = (a - \nu * I) * G, \qquad (7)$$

$$\frac{dL}{dt} = 2c^{0.5}(a - \nu * I) * L^{0.5} \qquad (8)$$

By the same logic, considering equation (4), one can say that an increase in the productivity gap G will lead to new investments, as a result of which the rate of decline in investment b will decrease by Δb, as a result of which we obtain the following equation for the dynamics of investment

$$\frac{dI}{dt} = (\Delta b - b) * I, \qquad (9)$$

The larger the performance gap G, the larger the Δb value, which in the linear approximation will look like this

$$\Delta b = \mu * G , \qquad (10)$$

where μ is the coefficient of proportionality.

Hence, to describe the dynamics of investment I, one can obtain the following differential equation

$$\frac{dI}{dt} = (\mu * G - b) * I. \qquad (11)$$

Combining equations (7) Ա (11) into one system, we get:

$$\left.\begin{aligned} \frac{dG}{dt} &= (a - \nu * I) * G, \qquad (12\text{-}1) \\ \frac{dI}{dt} &= (\mu * G - b) * I \qquad (12\text{-}2) \end{aligned}\right\}$$

The resulting system of differential equations (12) is a dynamic model of the predator-prey type of the productivity gap G and investment I aimed at eliminating this gap, where the role of the predator is played by investment, and the role of the prey is played by the gap in productivity.

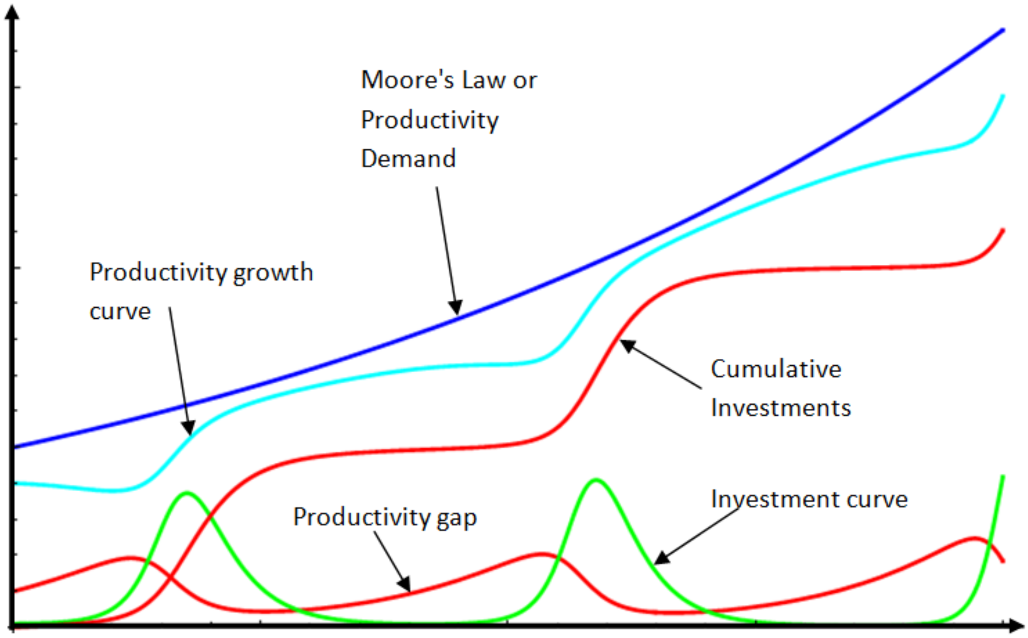

Fig. 1 Solutions to the system of equations (12) in the form of productivity gap and investment curves, which are mutually dependent and have a periodic nature

These equations can be used to study and manage investment processes in various spheres of human activity.

Fig. 1 shows the solutions to the system of equations (12) at the bottom of the image as productivity gap and investment curves in the area of development of Electronic Design Automation tools.

At the top of the image is Moore's Law [5], which shows the growth trend in productivity, below which is the growth curve of real productivity, and slightly below - the curve of total investment.

Using the equation of losses (8) instead of the productivity gap equation in the system of equations (12), one can obtain a new system of equations representing the dynamics of the loss-investment pair

$$\left.\begin{aligned} \frac{dL}{dt} &= 2c^{0.5}(a - \nu * I) * L^{0.5} \quad &(13\text{-}1) \\ \frac{dI}{dt} &= (\mu * G - b) * I \quad &(13\text{-}2) \end{aligned}\right\}$$

Solving the problem "Productivity gap vs. Investment" by the method of state equations

The main problem in obtaining in an analytical way analogs of systems of equations (12) or (13) by the method of state equations is the derivation of analogs of equations (1) and (4).

Let's first consider obtaining the productivity gap growth equation (1) by the method of state equations.

If a parameter of the productivity gap type is considered, then it is assumed that there is a real value P_r of this parameter and its maximum possible or desired value P_i.

This means that the same human system or workforce will have different equations of state for these two values of productivity.

In the case of real performance P_r, the equation of state will have the form [3, 4]

$$N * P_r * T = W * D \qquad (14)$$

The magnitude of activity W in this equation is the basis for the human system to generate various types of results [3, 4].

In this case, the human system will receive a profit C_r through the activity W, which in the case of a linear approximation will be

$$C_r = k * W , \qquad (15)$$

where k is the coefficient of proportionality.

Taking into account expressions (14) և (15), for the profit C_r one can obtain

$$C_r = \frac{k*N*P_r*T}{D} \qquad (16)$$

By analogy with this expression, for the profit at the maximum possible value of productivity P_i we will have

$$C_i = \frac{k*N*P_i*T}{D} \qquad (17)$$

From here we can get the dynamic versions of the static equations of state (16) և (17)

$$\frac{dC_i}{dt} = \frac{k*N*P_i}{D} \qquad (18)$$

and

$$\frac{dC_r}{dt} = \frac{k*N*P_r}{D} \qquad (19)$$

From these equations, using the value of the loss of profit $C_i - C_r$, it is possible to obtain a gap in productivity $P_i - P_r$ by subtracting equation (19) from equation (18) and as a result one can obtain

$$\frac{d(C_i - C_r)}{dt} = \frac{k*N}{D} * (P_i - P_r) \qquad (20)$$

In this equation, the $P_i - P_r$ value represents the productivity gap G, and the $C_i - C_r$ value is linearly related to the same productivity gap by the following simple expression:

$$C_i - C_r = k_\psi * G \qquad (21)$$

where k_ψ is the coefficient of proportionality.

Taking into account expression (21), from equation (20) one can obtain an analogue of the differential equation (1) of the productivity gap:

$$\frac{dG}{dt} = \frac{k*N}{k_\psi * D} * G \qquad (22)$$

In this equation, the coefficient $\frac{k*N}{k_\psi * D}$ is the growth rate of the productivity gap G, just like the coefficient a in equation (1).

This means that from equation (22) using expressions (6) it is possible to obtain the final form of the differential equation of the productivity gap in the same way as it was done for equation (7).

$$\frac{dG}{dt} = \left(\frac{k*N}{k_{\psi}*D} - \nu * I\right) * G, \quad (23)$$

Let us proceed to obtaining an analogue of the investment differential equation (11) by the method of state equations.

The meaning of the equation of state associated with investment is that the investor is a human system that has an equation of state that describes its activities.

$$N_I * P_I * T = W_I * D_I \quad (24)$$

In this equation, the meaning of the activity W_I is that it makes or generates investment, and the productivity P_I has the meaning of the intensity of the investment flow.

The creation or generation of investments using the W_I activity in the simplest case can be represented in the following linear form

$$I = k_I * W_I \quad (25)$$

At the next step, from equations (24) and (25), one can obtain a new equation of state for investment I

$$I = \frac{k_I * N_I * P_I * T}{D_I} \quad (26)$$

whose dynamic version is equivalent to the following differential equation

$$\frac{dI}{dt} = \frac{k_I * N_I * P_I}{D_I} \quad (27)$$

In this equation, the intensity P_I is determined by the intensities P_{IB} and P_{ID}, respectively, of the start and end of the investments.

$$P_I = P_{IB} - P_{ID} \quad (28)$$

If there is no productivity gap ($G = 0$), and, naturally, there will be no investments, i.e. $P_{IB} = 0$, as a result of which we will have

$$P_I = - P_{ID} \quad (29)$$

Ongoing investments in this situation will gradually end with intensity P_I, which in a simplest case will be linearly proportional to the current amount of investments I.

$$P_I = -k_{ID} * I\ , \quad (30)$$

where k_{ID} is the proportionality coefficient.

Substituting expression (29) into the equation of state (27), we obtain

$$\frac{dI}{dt} = -\frac{k_I * k_{ID} * N_I}{D_I} * I, \qquad (31)$$

which is the desired analogue of the differential equation (4).

In this equation, the coefficient $-\frac{k_I * k_{ID} * N_I}{D_I}$ is the rate of investment decline.

If the gap in productivity increases, new investments will appear, due to which the rate of decline in investments $-\frac{k_I * k_{ID} * N_I}{D_I}$ will decrease by the amount Δ, which in the linear approximation is determined by the expression

$$\Delta = \mu * G \qquad (32)$$

where μ is the coefficient of proportionality.

Thus, the equation of investment dynamics will take its final form.

$$\frac{dI}{dt} = (\mu * G - \frac{k_I * k_{ID} * N_I}{D_I}) * I, \qquad (33)$$

By combining differential equations (23) և (33) into the system, we obtain a dynamic mathematical model of a pair of productivity gap and investment.

$$\left.\begin{aligned} \frac{dG}{dt} &= (\frac{k * N}{k_\psi * D} - \nu * I) * G, \qquad (34\text{-}1) \\ \frac{dI}{dt} &= (\mu * G - \frac{k_I * k_{ID} * N_I}{D_I}) * I, \qquad (34\text{-}2) \end{aligned}\right\}$$

Mathematically, the system of equations (34), obtained on the basis of the equations of state, is similar to the system of equations (12), which is derived from completely different principles.

Qualitatively, the solution of equations (34) is also has the view shown in Fig.1.

An important difference between these two systems of equations is that the system of equations (34) contains a much larger number of coefficients, each of which has a specific physical meaning and is measurable, which is a great advantage over equations (12).

Discussion and conclusions

1. Mathematical models of the predator-prey type can have a large number of applications for the quantitative representation of the life and activity of all living systems, between which there are contradictions and conflicts.

2. One of the main goals of this series of works is to show that the method of equations of state, in addition to applications of a static nature, has a very wide range of dynamic applications.

3. The fact that two different approaches to modeling the dynamics of investment and the productivity gap lead to the same mathematical description suggests that the method of equations of state can have a much wider range of applications.

4. A quantitative representation of the behavior of predator-prey systems is characterized by the fact that it contains feedback that change from positive to negative and vice versa - a property that is easily described by dynamic equations of state.

5. In general, the main feature of the method of equations of state is that phenomena and processes, which by their nature are far from each other, can be described by the same equations of state, regardless of whether they relate to animal life or human activity and the reason for this is that in all cases these equations are based on the condition of a balance between the demands of life and the capabilities and skills of living systems.

6. Thus, in the first article of this series, such a balance is established between the requirements of life for predators and prey and their abilities, from which the well-known equations of the predator-prey model directly follow.

7. In this article, this balance again operates between the requirements of life and the capabilities and skills of people, but in this case it is more complex. The object of research here is the productivity of people, which in the process of development through investments moves from one paradigm to another, more effective paradigm.

8. In this sense, each paradigm has its maximum possibilities and potential for development, which is gradually depleted and this is expressed by the fact that a dynamic gap is created between the demands of life and the possibilities of people, the bridging of which requires new investments.

9. The third article in the series, devoted to the application of the method of equations of state in the field of geopolitics, discusses dynamic mathematical models of a unipolar and multipolar world order based on dynamic equations of state of superpowers.

10. Despite the fact that in order to facilitate the perception of the essence of the problems under study, the main attention is focused on periodic solutions of dynamic equations of state, but these equations also contain important aperiodic solutions, especially in the analysis of geopolitical problems.

References

1. Kristof Van Beeck, Filip Heylen, Jan Meel, Toon Goedem´e. (2010). Comparative study of Model-based hardware design tools.
file:///C:/Users/Pavel/Downloads/Comparative_study_of_Model-based_hardware_design_t.pdf

2. Murray J. D. (1993). Mathematical Biology. Second, Corrected Edition. Springer-Verlag Berlin Heidelberg.

3. Pavel Barseghyan (2017) "Elements of the Mathematical Theory of Human Systems Part 1: Assessment of the Results of Human Actions and Activities Based on the Method of State

Equations " *PM World Journal Volume 6, Issue 12 December 2017* – 13 pages.
http://pmworldjournal.net/wp-content/uploads/2017/12/pmwj65-Dec2017-Barsegyan-mathematical-equation-of-human-systems-part1-featured-paper.pdf.

4. Barseghyan, P. (2021). Quantitative description of the dynamics of interactions of living systems by the method of state equations – Part 1: Derivation of the equations of the predator-prey model from the equations of state of living systems; *PM World Journal*, Vol. X, Issue V, May. https://pmworldlibrary.net/wp-content/uploads/2021/05/pmwj105-May2021-Barseghyan-equations-of-the-predator-prey-model.pdf

5. Moore's Law. Wikipedia. https://en.wikipedia.org/wiki/Moore%27s_law.

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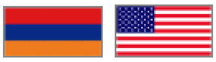

Dr. Pavel Barseghyan is a consultant in the field of quantitative project management, project data mining and organizational science. Has over 45 years' experience in academia, the electronics industry, the EDA industry and Project Management Research and tools development. During the period of 1999-2010 he was the Vice President of Research for Numetrics Management Systems. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of quantitative project management, mathematical theory of human work, electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia).

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