Quantitative description of the dynamics of interactions of living systems by the method of state equations

Part 3: The power of human systems and dynamic mathematical models of their interactions [1]

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Abstract

The main topic of this article is a quantitative presentation of the dynamics of the relationship between humans and human systems through mathematical models of the predator-prey type.

This representation of human relations is based on the assertion that each human system through its activity exerts positive and negative pressure on other human systems associated with it, which can be described by equations of state.

The article examines a dynamic mathematical model of the relationship between two human systems, which is a system of two differential equations reflecting the state of the system.

With the help of such a model of human interactions, it is possible to describe the victory and defeat of human systems in the form of aperiodic solutions of differential equations, as well as the balanced life of human systems without serious shocks in the form of periodic or quasiperiodic solutions of the same differential equations.

Introduction

The field of studying the interaction of human systems has traditionally been, and generally remains, the field of application of non-quantitative methods.

In the second half of the twentieth century, mainly under the influence of systems theory [1], cybernetics [2] and physics [3, 4], many attempts were made to quantify the problems of psychology, organizational science, project management, social and political sciences, geopolitical and other problems.

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A characteristic feature of all these approaches is that they all adapt the ideology, methods and approaches of other areas that have reached a high level of development to the study of the life and activities of people and human systems.

But as is known from the history of science, real scientific successes and achievements in any particular field become reality only when they are based on their own philosophy, ideology, methods and approaches, which can only be born on the basis of systematic research in that specific area.

In this sense, the quantitative science of life, behavior and activity of human systems is at that stage of its development when ideological and methodological borrowings from more developed fields of science have basically completed their work, on the basis of which it became possible to develop their own approaches to the problem.

The ideological basis of this series of articles in the field of mathematical modeling of the behavior and activity of biosystems is the principle of a fundamental nature according to which any biosystem in various situations seeks to ensure the continuity, longevity and perpetuation of its life [5, 6].

This principle is very rich in its essence, on the basis of which it is possible to explain the arbitrary manifestation of the course of animal life and the activity of human systems.

Since the continuity and longevity of life, in turn, are based on the safety of life, energy supply and its reproduction, this principle is divided into three sub-principles, namely:

- a sub-principle of energy security, including maximizing energy inflow and minimizing its consumption,

- a sub-principle of ensuring the safety of life with maximum probability,

- a sub-principle of ensuring the reproduction and training of the species with maximum probability.

These three sub-principles, in turn, can have many other branches, which can cover various phenomena and processes associated with the life of biological systems and the activities of human systems.

In particular, a sub-principle of this kind is Zipf's principle of minimum effort [7], which is a special case of the principle of minimum energy consumption, and which, among its many applications, can also cover linguistic problems [8].

In addition, the principle of minimum effort, in turn, can have its ramifications and consequences. In particular, in [9] quantitative universal laws, such as the Zipf-Pareto distributions, follow directly from the principle of minimum effort.

As for the dynamics of interactions of human systems, the mathematical representation of which by the method of equations of state is the goal of this article, it also goes back to the universal principle of the continuity and longevity of life.

In general, an adequate quantitative representation of the dynamics of interactions of human systems is based on two basic assumptions, based on which the differential equations of the dynamics of biosystems are derived from their equations of state.

Before proceeding to the mathematical description of the problem, let us consider the relationship between the equations of state of human systems and two basic statements representing the behavior of interacting people with the principle of continuity and longevity of life.

The life path of any living system is a sequence of steps, decisions and actions necessary to continue the normal course of life [10].

That is, every time life makes its next demand, and a living system or human system reacts to this demand in accordance with its capabilities and abilities and fulfills it, achieving some result.

If the life of the human system continues to remain normal, this means that there is a certain correspondence between the requirements of life, on the one hand, and the abilities of people, on the other, the quantitative expression of which is its equations of state [10].

That is, the equations of state of living systems, and in particular, human systems, are a direct quantitative consequence of the principle of continuity and longevity of life.

When it comes to the process of interaction of biosystems, both people and animals in this process receive benefits and losses associated with their physical and energy security and their reproduction [5].

For example, for a predator, the energy security of life associated with the requirement for the continuity and longevity of life means getting the maximum amount of food as a source of energy (maximum benefit) and spending the least amount of energy (minimum loss) to ensure physical and energy security.

Let us now consider the behavior of human systems in certain specific situations, in the process of their interaction.

In general, the process of interaction of human systems includes actions associated with their confrontation and cooperation.

In the case of a regime of confrontation between human systems, they put pressure on the other side in order to gain an advantage over it and have the greatest generalized benefit from this victory, and in the case of cooperation, they create a regime of incentives for each other, which is quantitatively equivalent to putting negative pressure on each other the goal of which, as always, is to obtain the maximum generalized benefit under specific conditions.

Suppose we have two interacting human systems that are in a balanced conflict with each other, that is, the pressure exerted by the first system on the second is, on average, equal to the pressure exerted by the second system on the first.

Under these conditions, let's conduct a thought experiment, assuming that the pressure of the second system on the first human system for some reason has dropped sharply, that is, an unbalanced state is created when the pressure of the first human system on the second. much more than the pressure of the opposite side.

An abrupt change in the balance of power will force the first party to make a decision that is appropriate for the situation, increasing or decreasing its pressure on the second human system.

The choice of any of these solutions will depend on additional circumstances associated with an increase in the expected interest or a decrease in the losses of the first side of the confrontation.

Under these conditions, the principle of continuity and longevity of life will force the first human system to choose a solution that will lead to the maximum benefit for itself, which, in turn, will depend on what material and non-material potential sources of benefit the weakening second side has.

Hypothesis 1. If the weakening second party to the conflict has significant potential sources of interest, then the first human system will sharply increase its pressure on the adversary, assuming that from this it will have much greater benefits in the form of territory and wealth compared to the inevitable losses associated with such a decision.

Hypothesis 2: If the weakening side of the conflict does not have potential sources of interest, then the first human system must choose the second solution, which reduces the pressure on the weakened side, because maintaining pressure without expectations will be associated with meaningless losses.

The two idealized extreme cases discussed above in relation to human systems will serve as a conceptual basis for a mathematical description of the dynamics of interactions between human systems.

Mathematical models of the dynamics of mutual pressures between human systems

The life of an arbitrary human system is an interconnected process of various types of activities.

In turn, each activity of an arbitrary human system is a chain of actions pursuing certain goals, which has its own equation of state [11, 12].

Thus, according to the method of equations of state, the mathematical model of the life cycle of each human system is a system of interconnected equations of state, the number of which is equal to the number of activities of this human system.

In addition, each type of human activity can be represented as the sum of its sub-activities, each of which, as a sub-chain of certain types of actions, has its own equation of state as well.

With this approach, increasing the number of equations representing the state of life of a human system, one can describe its life in all details, including connections and interactions with other human systems.

Since relations between people, like relations between large-scale human systems, can be represented as a sequence of actions and as a certain type of activity of human systems, they can also have equations of their state.

Suppose we are considering two human systems, which, as part of their activity in the form of a sequence of actions, exert pressure on each other [11, 12].

According to the method of equations of state, this means that each side has its own equation of state, which determines their pressure on the opposite side.

The equation of state for the first side over a finite period of time T will look like this

$$N_{12} * P_{12} * T = W_{12} * D_{12} , \qquad (1)$$

and for the second side of confrontation, the equation of state will be

$$N_{21} * P_{21} * T = W_{21} * D_{21} , \qquad (2)$$

These two equations are symmetrically similar to each other and have the same meaning, which for the first side will sound as follows.

The first side of the confrontation with the number of people N_{12}, who have an average productivity P_{12} (in terms of creating pressure on the opposite side), during the period T perform the activity W_{12}, as a result of which it exerts pressure Q_{12} on the second side, overcoming the difficulty or resistance created by the second side D_{12} .

Since the pressure Q_{12} is created by the activity of W_{12}, then, in the first approximation, the linear relationship between these two quantities will look like this

$$Q_{12} = k_{12} * W_{12} , \qquad (3)$$

where k_{12}-is the coefficient of proportionality, which is the pressure created by the unit of activity of the first side on the second side of the confrontation.

Taking into account expression (3), from the equation of state (1) we obtain a new equation in which the pressure Q_{12} also participates.

$$k_{12} * N_{12} * P_{12} * T = Q_{12} * D_{12} \,, \qquad (4)$$

The dynamic version of this equation for the generation or creation by the first side of confrontation on the second side pressure ΔQ_{12} during the period Δt will be.

$$k_{12} * N_{12} * P_{12} * \Delta t = \Delta Q_{12} * D_{12} \,, \qquad (5)$$

which is equivalent to the following differential equation

$$\frac{dQ_{12}}{dt} = \frac{k_{12}*N_{12}*P_{12}}{D_{12}} \qquad (6)$$

Symmetrically, a similar differential equation is also satisfied by the pressure Q_{21} of the second side of the conflict on the first.

$$\frac{dQ_{21}}{dt} = \frac{k_{21}*N_{21}*P_{21}}{D_{21}} \qquad (7)$$

Dynamics of mutual pressure of the conflicting parties in different situations

Since the interaction of the conflicting parties occurs in the form of a sequence of actions, in this sense, the productivity of each of the parties P_{12} and P_{21} is the intensity of actions that put pressure on the other side of the confrontation.

In different situations, there can be actions between the conflicting parties that contribute to increasing pressure on the other side, and, conversely, actions that help reduce confrontation or mutual pressure.

In this sense, each performance P_{12} և P_{21} is the sum of two streams of actions.

$$P_{12} = P_{12+} + P_{12-} \qquad (8)$$

and

$$P_{21} = P_{21+} + P_{21-} \qquad (9)$$

Substituting expressions (8) և (9) into equations (6) և (7), we obtain:

$$\frac{dQ_{12}}{dt} = \frac{k_{12}*N_{12}*(P_{12+}-P_{12-})}{D_{12}} \qquad (10)$$

$$\frac{dQ_{21}}{dt} = \frac{k_{21}*N_{21}*(P_{21+}-P_{21-})}{D_{21}} \qquad (11)$$

Comparison of equations (10) and (11) with the situations presented in the above hypotheses allows one to draw qualitative and quantitative conclusions about the behavior of the conflicting parties.

Let us continue the analysis by transforming equation (10) for the pressure created by the first side, in accordance with the situation presented in the first hypothesis.

Mathematical model of the behavior of the first side of confrontation

If the second side of the confrontation for some reason weakened and does not resist the first side and, in addition, is of serious interest as a source of benefit, then the first side, according to Hypothesis 1, no longer needs actions aimed at weakening the confrontation and, therefore, $P_{12-} = 0$.

Hence, for equation (10), we obtain

$$\frac{dQ_{12}}{dt} = \frac{k_{12} * N_{12} * P_{12+}}{D_{12}} \qquad (12)$$

In this equation, the value $N_{12} * P_{12+}$ is the power U_{12} of the first side of the confrontation.

$$U_{12} = N_{12} * P_{12+} \qquad (13)$$

On the other hand, there is a direct relationship between the pressure Q_{12} and the power U_{12}, which for a linear approximation will have the following form

$$Q_{12} = h_{12} * U_{12} \qquad (14)$$

where h_{12} is the proportionality coefficient.

Taking into account expressions (13) և (14), for equation (12) we obtain

$$\frac{dQ_{12}}{dt} = \frac{k_{12} * Q_{12}}{h_{12} * D_{12}} \qquad (15)$$

In general, this equation is a mathematical model that reflects the behavior of the human system to exert pressure on others for profit or simply robbery in a situation where there is no resistance from the oppressed parties.

In this sense, equation (15) is one of the direct consequences of the principle of continuity and longevity of life and, in other words, it is a mathematical model of an uncontrolled increase in pressure on the losing side in conflicts.

This equation has an increasing exponential solution, where the value $\frac{k_{12}}{h_{12}*D_{12}}$ is the rate of increase of the pressure Q_{12}.

Naturally, an excessive increase in pressure Q_{12} should increase the resistance of the other side in the form of an increase in pressure Q_{21}, as a result of which the rate $\frac{k_{12}}{h_{12}*D_{12}}$ of increase in pressure Q_{12} will decrease by some amount Δ_{21}.

Naturally, the value Δ_{21} will be related to the pressure Q_{21}, which can be represented as a linear approximation as follows:

$$\Delta_{21} = \beta_{21} * Q_{21} \qquad (16)$$

where β_{21} is the coefficient of proportionality.

Hence, we obtain a mathematical model of the pressure of the first side of the conflict on the second side in the form of the following differential equation

$$\frac{dQ_{12}}{dt} = (\frac{k_{12}*Q_{12}}{\mathrm{h}_{12}*D_{12}} - \beta_{21} * Q_{21}) * Q_{12} \qquad (17)$$

Mathematical model of the behavior of the conflicting second party

The next important situation in which one of the parties to the conflict, in this case the second side, may find itself, will be when, for some reason, the first side of the confrontation ceases to exert pressure on the other side.

In this situation, due to the fact that the behavior of the second side of the confrontation is also guided by the principle of continuity and longevity of life, from which follows a number of requirements related to the minimum expenditure of energy, finances, efforts and risks, it will begin to weaken its Q_{21} pressure on the first side.

In other words, applying Hypothesis 2 to the equation of state (11) of the conflicting second side, we can see that if, for some reason, the first side stops its pressure on the second side, but at the same time, the first side cannot be a potential source of benefit for the second side, then for the latter it makes no sense to increase the pressure on the first side.

Quantitatively, this means that in this situation in the equation of state (11) $P_{21+} = 0$, as a result of which we obtain

$$\frac{dQ_{21}}{dt} = -\frac{k_{21}*N_{21}*P_{21-}}{D_{21}} \qquad (18)$$

In this equation, as in the previous case, the value $N_{21} * P_{21-}$ is the power U_{21} of the second side of the conflict.

$$U_{21} = N_{21} * P_{21-} \qquad (19)$$

Since in this case there is also a direct relationship between the pressure Q_{21} and the power U_{21}, then in the linear approximation we will have

$$Q_{21} = h_{21} * U_{21} \qquad (20)$$

where h_{21} is a coefficient of proportionality.

Taking into account expressions (19) և (20), for equation (18) we obtain

$$\frac{dQ_{21}}{dt} = -\frac{k_{21}*Q_{21}}{h_{21}*D_{21}} \qquad (21)$$

This equation is a mathematical expression of the statement that if the pressure exerted on any side in human relations leads only to losses, it is meaningless and must be stopped.

It is this statement that reflects equation (21), which has a decreasing exponential solution, and which describes the inertial process of reducing costs and risks caused by past tensions between the parties to the conflict. In this solution, the value $\frac{k_{21}}{h_{21}*D_{21}}$ is the rate of pressure decrease on the first side of the conflict.

But in real life there is always a conflict of interest and an excessive weakening of the pressure of Q_{21} will lead to an increase in the pressure of Q_{12} of the first side of the confrontation on the second side.

Naturally, an increase in pressure Q_{12} should reduce the rate $\frac{k_{21}}{h_{21}*D_{21}}$ of the decrease in pressure Q_{21} of the second side by some value Δ_{12}, which will directly depend on the value of the pressure Q_{12}, and which can be represented as a linear approximation as follows:

$$\Delta_{12} = \beta_{12} * Q_{12} \qquad (22)$$

where β_{12} is a coefficient of proportionality.

From here, taking into account equation (21) and expression (22), we obtain a mathematical model of the pressure of the second side of the conflict with respect to the first side in the form of the following differential equation.

$$\frac{dQ_{21}}{dt} = (\beta_{12} * Q_{12} - \frac{k_{21}*Q_{21}}{h_{21}*D_{21}}) * Q_{21} \qquad (23)$$

Dynamic mathematical model of confrontation of human systems

Combining equations (17) և (23), representing the dynamic behavior of the conflicting parties, into one system, we obtain

$$\frac{dQ_{12}}{dt} = (\frac{k_{12}*Q_{12}}{h_{21_{12}}*D_{12}} - \beta_{21} * Q_{21}) * Q_{12} \qquad (24\text{-}1)$$

$$\frac{dQ_{21}}{dt} = (\beta_{12} * Q_{12} - \frac{k_{21}*Q_{21}}{h_{21}*D_{21}}) * Q_{21} \qquad (24\text{-}2)$$

This system of differential equations is similar in structure to the mathematical model of the predator-prey [13], where the analogue of the prey is the first side of the confrontation, and the analogue of the predator is the second side of the confrontation.

But there is also a significant difference between the model of confrontation of human systems (24) and the classical mathematical model of the Lotka-Volterra predator-prey, which is that in the case of human systems, the model is completely symmetric with respect to the confronting sides, and the predator-prey model in this sense asymmetric, since it is impossible to swap the places of the predator and prey.

These two models are similar in that the ways and means by which the classical predator-prey model was developed and enriched [13] are also applicable to the description of the dynamics of human relations.

In particular, if states or their coalitions are viewed as human systems, the models of intraspecific competition used in the predator-prey model are very important for an adequate quantitative representation of intrastate competition of centers of power and for interstate relations within coalitions.

The equations representing the dynamics of the behavior of human systems (24) are mathematically well studied [14] and have periodic and aperiodic solutions that can describe the dynamics of conflicts and cooperation between people.

Conclusions

1. The most important conclusion that can be drawn from this work is that the method of equations of state allows a new approach to the problem of quantitatively describing the interactions of human systems.

2. The relationship between human systems is based on the principle of continuity and longevity of life, on the basis of the direct consequences of which it is possible to build mathematical models that adequately reflect human behavior in various situations.

3. In particular, this refers to the desire of human systems to maximize generalized benefits and minimize generalized losses and their implementation as direct consequences of the principle of continuity and longevity of life.

4. The same principle justifies the behavior of people and large-scale human systems in conflict situations, when not meeting resistance, they maximize the pressure on the opponent in order to obtain the maximum generalized benefit.

5. In such a situation, when in the course of the conflict the opposing side again does not offer resistance, but at the same time is not a source of interest for the winning side, on the basis of the same principle, the winning side reduces its pressure on the other side in order to avoid unnecessary losses as much as possible.

6. Moreover, the losing side in the conflict will begin to resist only when it sees that its losses in non-resistance are greater than in resistance, behavior that also corresponds to the principle of continuity and longevity of life.

7. Through the principle of continuity of life, one can also quantitatively describe the dynamics of coalition formation at the state level, the dynamics of interactions between centers of power, to which the fourth part of this series of articles will be devoted.

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 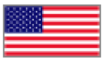

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